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Statistics and Evolution of Stellar Magnetic Fields and Magnetic Fluxes

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Abstract. Statistical properties of *rms* mean magnetic fields $\mathcal{B}$ of normal stars were investigated based on data from known catalogues of magnetic fields and recent measurements. We study the magnetic field distribution function and find that it has a steep drop in a weak magnetic field region $\mathcal{B} < 400$ G. This drop may be connected with a rapid dissipation of the weak stellar magnetic field proposed by Auriere et al. (2007). We have estimated the magnetic fluxes $\mathcal{F}$ of all stars with measured magnetic fields and analyze their statistical properties.

1 Introduction

At the present time the magnetic fields of more than a thousand normal stars have been detected (Bychkov et al., 2009). The field and its structure strongly depend on the stellar mass. Relatively low–mass convective stars with $M < 1.5 - 2\,M_\odot$ possess the magnetic fields that are mainly small–scale and variable with time, which are connected with the dynamo action driven by a differential rotation and convection, or buoyancy instabilities.

The stars in the upper main-sequence (MS) with $M > 2\,M_\odot$ have an extended radiative envelope and a small convective core. These more massive stars without convective envelopes tend to have large–scale, steady magnetic fields at least as long as they have been observed since the first magnetic field detection by Babcock (1947).

Most of these stars have roughly dipolar fields (e. g., Auriere et al., 2007), but some of them possess more complex fields, such as the star τ Sco with a surprising magnetic topology (Donati et al., 2006). The white dwarfs are the remnants of low and intermediate–mass stars. Magnetic white dwarfs have total magnetic fluxes close to the fluxes of their progenitor stars (e. g., Ferrario & Wickramasinghe, 2005).

More massive OB stars are the progenitors of neutron stars, the fields of which often have a dipole geometry (Annala & Poutanen, 2010). Both OB and less massive A stars are non–convective. It means that the dynamo–mechanism is not effective for these stars and it can be the argument in favor of the hypothesis that the magnetic fields of these stars are the *fossil* remnants in some stable equilibrium configuration.

In recent papers by Braithwaite & Nordlund (2006) and Braithwaite (2008) it was shown by numerical simulations that stable magnetic field configurations do exist under the conditions of the radiative interior of OBA stars. These results confirm a hypothesis by Prendergast (1956) that the balanced magnetic field configuration has poloidal and toroidal fields of roughly the same strength. Braithwaite (2008) finds that the tori of twisted fields can be formed as the remnants of decay of an unstable random initial field. In agreement with observations, the surface fields are roughly dipole with smaller contributions of higher multipoles, and the surface field strength can decrease with age of the star.

Nevertheless, some investigators suppose that the role of dynamo action can be significant. Recently Augustson et al. (2010) have investigated the role of core convection in the generation of magnetic fields inside then B stars. They employ a 3D anelastic spherical harmonic (ASH) code to the model of turbulent dynamics within a 10 $M_\odot$ MS B–type star with a rotation velocity of $\Omega = 4\Omega_\odot$. They find that strong (900 kG) magnetic fields arise within the turbulence of the core, and penetrate into the stably stratified radiative zone. These fields exhibit a complex, time–dependent behavior including the reversals in magnetic polarity and change between the hemispheres dominating in the total magnetic energy.

In order to improve our knowledge of magnetic fields of normal stars, we have undertaken a statistical study of a sample of all the stars with measured magnetic field. Our principal attention is focused to the OBA stars with more or less stable fields, since strong variations of the mean magnetic fields in the stars of later spectral classes make the statistical studies of their magnetic fields very problematic.

We present a description of our sample of magnetic fields in Sect. 2. An analysis of the magnetic field functions is presented in Sect. 3. In Sect. 4 we describe the results of calculation of stellar magnetic fluxes and analyze the magnetic flux distributions in various kinds of stars. In Sect. 5 we discuss our results, and, finally, some conclusions are laid out in Sect. 6.

2 Magnetic Field Catalogue

Our sample of magnetic fields for normal stars consists mainly of the data presented in the catalogue by Bychkov et al. (2009) and its previous version Bychkov et al. (2003). We also add to these data new measurements of magnetic fields from Bouret et al. (2008), Hubrig et al. (2008, 2009a, 2009b), Kholtygin et al. (2007), Petit et al. (2008), Schnerr et al. (2008), McSwain (2008) and Silvester et al. (2010).

As a statistical measure of a magnetic field we use the *rms* magnetic field

$$\mathcal{B} = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(\overline{B}_l^i)^2}\,, \tag{1}$$

where we make a sum over all the measured values of effective magnetic fields $\overline{B}_l$, and n is a number of observations. In the paper by Kholtygin et al. (2010a) it was shown that in the case of the dipole field configuration the *rms* field $\mathcal{B}$ weakly depends on the random values of the rotational phases ϕ of observations, the dipole inclination angle i and the angle β between the rotational axis and that of magnetic dipole. This conclusion stands for quadrupole and other field configurations. Using other statistical characteristics of the stellar magnetic field (Kholtygin et al., 2010a) is also possible.

As a measure of accuracy of magnetic field measurements, the value $\Sigma_\mathcal{B}$ is normally used:

$$\Sigma_\mathcal{B} = \sqrt{\frac{1}{n}\sum_{i=1}^{n}\left(\sigma_{B_l^i}\right)^2}\,. \tag{2}$$

Here $\sigma_{B_l^i}$ is the *rms* error of field measurement i. The widely used value of $\Sigma_\mathcal{B}$ is not a standard deviation of the *rms* field $\mathcal{B}$, but however it can be considered as an estimation of accuracy of the measured magnetic field.

To estimate if the field measurements are real, the reduced chi–square statistics, where

$$\chi^2/n = \frac{1}{n}\sum_{i=1}^{n}\frac{\left(B_l^i\right)^2}{\left(\sigma_{B_l^i}\right)^2}\,. \tag{3}$$

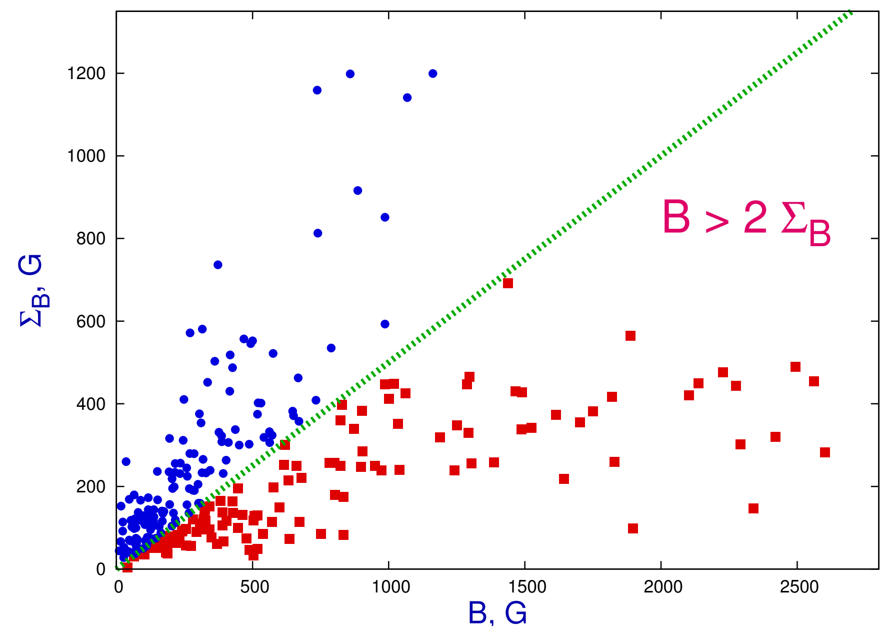

Figure 1: Stars with measured magnetic fields on the plane $B-\Sigma_{\rm B}$ for $B \le 2.7\,$kG and $\Sigma_{\rm B} \le 1.35\,$kG (triangles). The thick dotted line marks the dependence $B = 2\Sigma_B$. The measurements which satisfy the conditions (4) are marked with filled red squares, the others with blue points.

is also used.

As a measure of reality of the fields measured we will use the following criteria (see for details Kholtygin et al., 2010a):

$$\left\{ \begin{array}{lcr} \mathcal{B} & > & 2\,\Sigma_{\mathcal{B}}\,, \\ \chi^2/n & > & 1\,. \end{array} \right. \tag{4}$$

In Fig. 1 we plot all the B stars from the catalogue by Bychkov et al. (2009) in the plane $B-\Sigma_{\rm B}$ to illustrate the procedure of selection of real magnetic fields.

In Fig. 2 we present the *rms* magnetic field values $\mathcal{B}_{\rm mean}$ averaged over different spectral classes. In the figure one can see a large jump of the mean magnetic fields between the O and B spectral classes as it was first reported by Kholtygin et al. (2010a, 2010b). Supposing that the magnetic fluxes of O and B stars are similar, we can explain the difference by the larger radii of O stars. It is also possible that this effect can be linked with magnetic flux loss in O stars due to their powerful stellar winds.

Starting with stars of the spectral class B, the mean magnetic field regularly decreases to later spectral types. One may link it with the fact that the lower the stellar mass is, the greater is a contribution of the irregular field to the total stellar magnetic field. We have to note that for the F and later stars the mean magnetic field values presented in Fig. 2 are the upper limits.

3 Magnetic Field Function

The analysis of the differential magnetic field distribution $f(\mathcal{B})$ (the magnetic field function) introduced by Bychkov et al. (1997) is important for understanding the origin of stellar magnetic fields. The function $f(\mathcal{B})$ is defined as follows:

$$N(\mathcal{B},\mathcal{B}+\Delta\mathcal{B}) \approx N f(\mathcal{B}) \Delta\mathcal{B}\,, \tag{5}$$

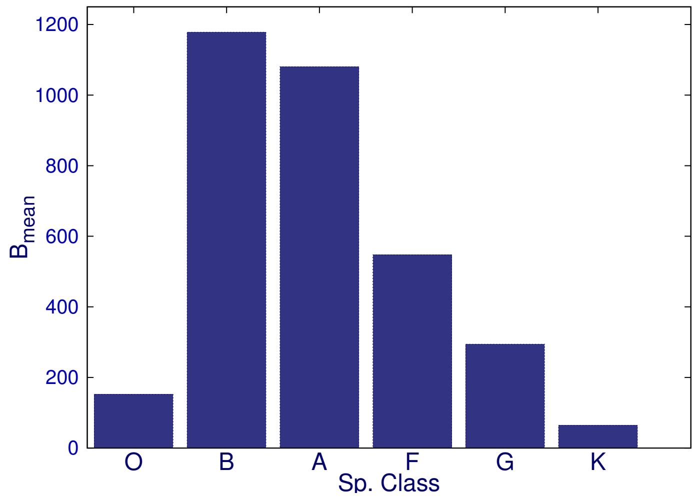

Figure 2: Magnetic fields of normal stars averaged over spectral classes.

where $N(\mathcal{B},\mathcal{B}+\Delta\mathcal{B})$ is the number of stars in the interval of mean magnetic fields $(\mathcal{B},\mathcal{B}+\Delta\mathcal{B})$, N is the total number of stars with measured field $\mathcal{B}$ satisfying the condition (4). In this paper, we restrict ourselves only to B and A stars, because the number of O stars with measured magnetic fields is insufficient for statistical studies and it is impossible to obtain the magnetic field function for O stars.

After using the criteria (4) we have 174 B stars with statistically significant fields. The magnetic field distribution function $f(\mathcal{B})$ we obtained for B stars is shown in Fig. 3. We choose the bins of the mean magnetic fields with at least eight stars in each bin. Only in the regions $\mathcal{B}<0.06$ kG and $\mathcal{B}>5$ kG the number of stars in the intervals appear to be smaller than eight, since the number of stars with very low and very strong magnetic fields is small.

The derived function $f(\mathcal{B})$ for $\mathcal{B}\geq\mathcal{B}^{\mathrm{tr}}$ can be fitted with a power law:

$$f(\mathcal{B}) = A_0\left(\frac{\mathcal{B}}{\mathcal{B}_0}\right)^{\gamma}. \qquad (6)$$

Here $\mathcal{B}^{\mathrm{tr}}$ is the threshold value of the *rms* field, which is determined by the shape of the function $f(\mathcal{B})$. It turned out that in a wide range of values of $\mathcal{B}$ $(0.40-12\,\mathrm{kG})$, the distribution function $f(\mathcal{B})$ for B stars could be described with an expression (6) with the parameters $\mathcal{B}_0=1\,\mathrm{kG}$, $\mathcal{B}^{\mathrm{tr}}=400\,\mathrm{G}$, $A_0=0.34\pm0.06$, and $\gamma=2.09\pm0.13$, as it is shown in Fig. 3.

Using the same criteria (4) for the A stars, we find 206 stars with statistically significant fields. The calculated magnetic field function $f(\mathcal{B})$ for the A stars is plotted in Fig. 4. We choose the bins of the *rms* mean magnetic fields for the A stars in the same way as it was done for the B stars.

The distribution $f(\mathcal{B})$ for the A stars could also be fitted by the expression (6) with the same $\mathcal{B}_0$ and $\mathcal{B}^{\mathrm{tr}}$ as for B stars, but with $A_0=0.36\pm0.13$ and $\gamma=2.37\pm0.30$, as shown in Fig. 4.

The catalogue of magnetic fields of CP stars is presented by Romanyuk & Kudryavtsev (2008), containing 355 B and A stars. Magnetic fields of 282 objects from this catalogue satisfy the criteria (4). The magnetic field function constructed from the data of the catalogue by Romanyuk & Kudryavtsev (2008) can also be described by a relation (6). The fit parameters are $A_0=0.37\pm0.06$ and $\gamma=1.80\pm0.19$. Note that the parameters of the fit obtained from the data of Bychkov et al. (2009) and Romanyuk & Kudryavtsev (2008) are similar within their errors although these catalogues do not contain the same stars.

We can conclude that a unified magnetic field function at least for early–type magnetic stars does exist. As we can see, the parameters of the magnetic field distribution of the B and A stars are

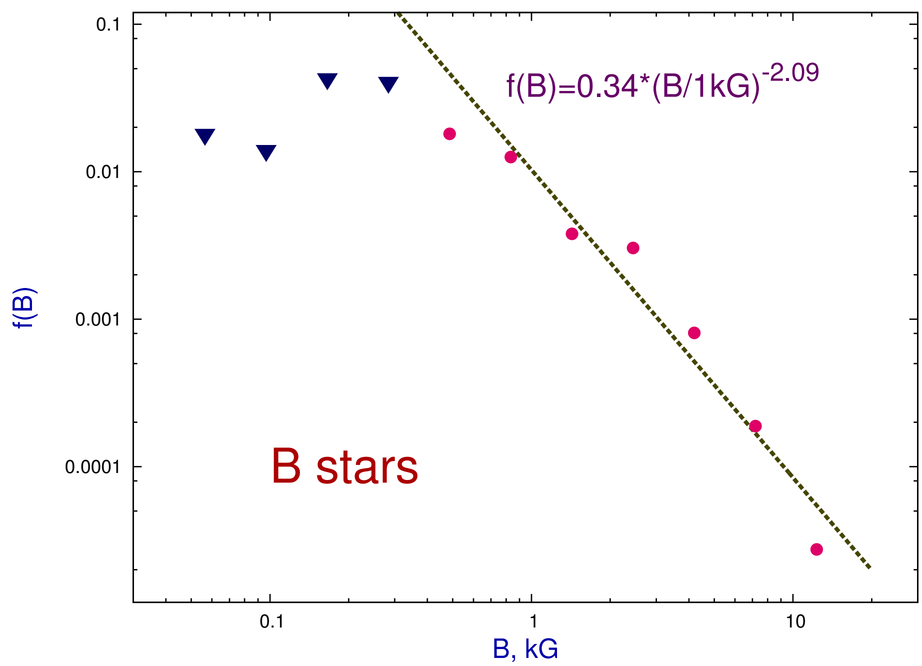

Figure 3: Magnetic field function for B stars (points and triangles). Its power fit (6) is shown with the thick pointed line. The triangles mark the data in the $\mathcal{B} < \mathcal{B}^{\rm tr}$ region.

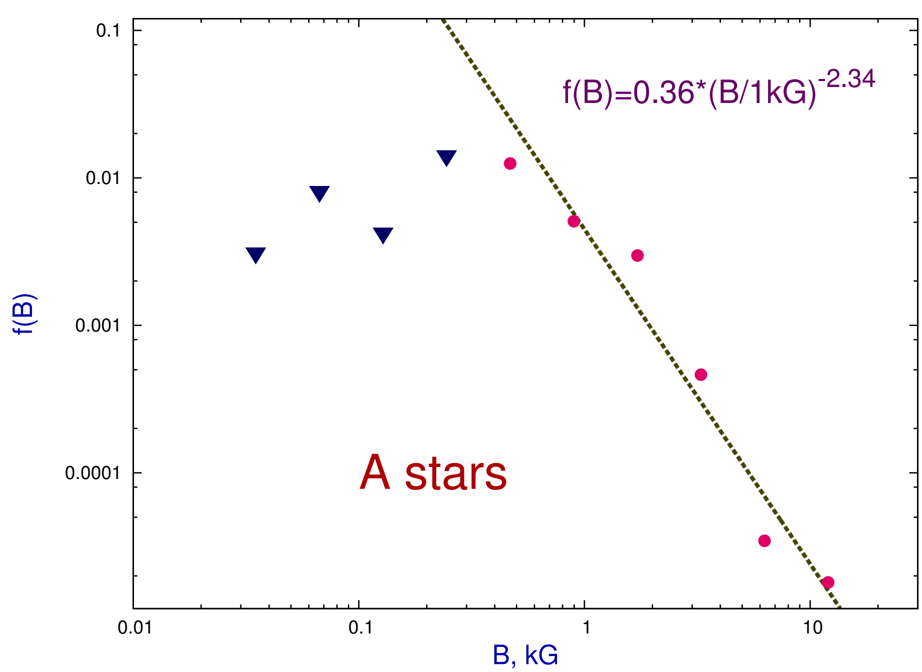

Figure 4: The same as in Fig. 3, but for A stars

similar. This means that a common mechanism of formation and destruction of the magnetic field of these groups of stars does exist.

Monin et al. (2002) constructed a magnetic field function for the surface magnetic field B_s from a sample of 57 bright ($V<4.0^m$) main sequence magnetic B3 – F9 stars.

From papers by Kholtygin et al. (2010a) and Monin et al. (2002) it is easy to conclude that a value of B_s is a factor of ≈ 4 larger than the *rms* magnetic field $\mathcal{B}$. The magnetic field function was fitted by Monin et al. (2002) with a power law (6).

For $B_s > 1.6$ kG ($\mathcal{B} > 400$ G), the authors Monin et al. (2002) obtained a value of $\gamma = 2.2$, which is close to our values for the A and B stars. In the range of magnetic fields $B_s = 0.4 - 1.6$ kG ($\mathcal{B} = 100 - 400$ G)) Monin et al. (2002) obtained a value of $\gamma \approx 1$ and concluded that there is a break in the magnetic field function in this range. Our data do not contradict the conclusion about the existence of such a break (see Figs.3 and 4). However, since the number of stars with measured magnetic fields in the above range is relatively small, the value of γ in the region of $\mathcal{B} < 0.4$ kG can not be found accurately.

3.1 Restoring the Real Magnetic Field Function

The behavior of the function $f(\mathcal{B})$ at relatively low values of $\mathcal{B} \leq 0.4$ kG is of particular interest. The corresponding values of $f(\mathcal{B})$ are indicated in Figs. 3 and 4 by the filled triangles. At such values of $\mathcal{B}$, the behavior of $f(\mathcal{B})$ does not obey the dependence (6). At small *rms* magnetic fields $\mathcal{B} < 0.1$ kG the value of $f(\mathcal{B})$ is lower than that obtained from the fit (6) by more than an order of magnitude. We see in Figs. 3 and 4 that the empirical magnetic field function changes its slope at $\mathcal{B}$ below the threshold value of $\mathcal{B}^{\mathrm{tr}} < 0.4$ kG.

The deviations $f(\mathcal{B})$ from (6) may be due to two reasons: low probability of detecting relatively weak magnetic fields (the observational selection), and a real change in the magnetic field function for relatively small fields. It is also probable that these two reasons are co–active.

We consider first the role of a low probability of detecting the weak fields $\mathcal{B}$. Following Kholtygin et al. (2010a) we introduce the field detection probability $P(n, \sigma_{\mathcal{B}}, \mathcal{B}, k)$. This value is determined as a probability to measure the magnetic field of stars with the *rms* field $\mathcal{B}$, if n field measurements were made. For the sake of simplicity it is proposed that all the measurements were made with an ideal spectropolarimeter that measures the field with an accuracy of $\sigma_{\mathcal{B}}$. The field will be detected if at least $k \geq 1$ measured values with $\mathcal{B} > 3\sigma_{\mathcal{B}}$.

The use of the Monte Carlo method simulations to calculate the probability $P(n, \sigma_{\mathcal{B}}, \mathcal{B}, k)$ was outlined by Kholtygin et al. (2010a) and Kholtygin et al. (2010b). It appears that for $n \geq 3$ this probability depends very weakly on n itself. Kholtygin et al. (2010a) argued that the value of the parameter $k = 2$ has to be used. In this case $P(n, \sigma_{\mathcal{B}}, \mathcal{B}, k) = P(n, \sigma_{\mathcal{B}}, \mathcal{B}, 2) \approx P(\sigma_{\mathcal{B}}, \mathcal{B})$. Extensive calculations of $P(\sigma_{\mathcal{B}}, \mathcal{B})$ were made by Kholtygin et al. (2010b).

The probability to detect the magnetic field depending on $\sigma_{\mathcal{B}}$ is plotted in Fig. 5. We can see a sharp decrease of the probabilities $P(\sigma_{\mathcal{B}}, \mathcal{B})$ for $B \leq 2\sigma_{\mathcal{B}}$.

Secondly, we may assume that for low values of $\mathcal{B}$ the magnetic field function can not be described by the power law with an index γ, obtained for large values of $\mathcal{B}$. Following Monin et al. (2002) we may suppose that the slope γ_{c} of the magnetic field function for the low values of $\mathcal{B} < \mathcal{B}_{\mathrm{c}}$ is other than γ. Here $\mathcal{B}_{\mathrm{c}}$ is a minimal value of $\mathcal{B}$, for which the power law fit of the magnetic field function is still valid. The Figs.3 and 4 show that $\gamma_{\mathrm{c}} < \gamma$.

For the sake of simplicity we suppose that $\gamma_{\mathrm{c}} \approx 0$. In this case, the total magnetic field function may be written as follows:

$$f(\mathcal{B}) = \begin{cases} A_0 \mathcal{B}_{\mathrm{c}}^{-\gamma} & , \mathcal{B} < \mathcal{B}_{\mathrm{c}} \, , \\ A_0 \mathcal{B}^{-\gamma} & , \mathcal{B} \geq \mathcal{B}_{\mathrm{c}} \, . \end{cases} \tag{7}$$

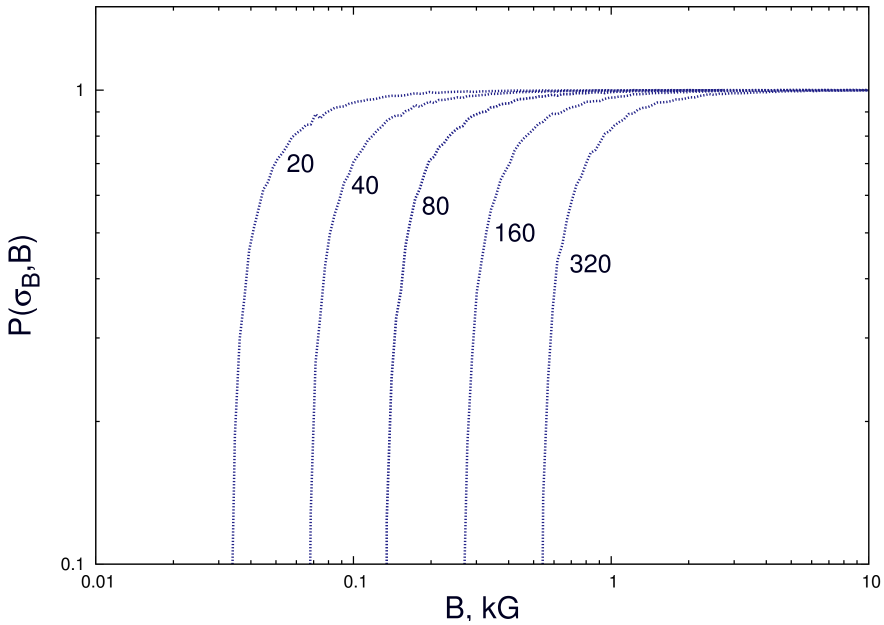

Figure 5: The probabilities of $P(\sigma_{\mathcal{B}},\mathcal{B})$ to detect the magnetic field for different values of $\sigma_{\mathcal{B}}$

Here we assume that $\mathcal{B}_0 = 1\,$kG. It means that hereinafter the value of $\mathcal{B}$ will be expressed in kG. For the sake of simplicity we leave the designation $\mathcal{B}$ for the *rms* magnetic field unchanged.

According to the relation (5) the function $f(\mathcal{B})$ is normalized:

$$\int_0^\infty f(\mathcal{B})d\mathcal{B} = 1\,. \tag{8}$$

From (8) it is easy to obtain

$$A_0 = \frac{\gamma - 1}{\gamma} B_{\rm c}^{\gamma-1}\,. \tag{9}$$

If the value of $\mathcal{B}_{\rm c} < \mathcal{B}^{\rm tr}$, then a large fraction of magnetic stars is undetectable. It follows that the total number of stars of the studied group (as an example, for the stars of the selected spectral class) can be presented by the following expression:

$$N_{\rm tot} = \mathcal{Q}\cdot\mathcal{R}\cdot N_{\rm meas} \quad = \quad \mathcal{Q}\cdot\gamma\left(\frac{\mathcal{B}^{\rm tr}}{\mathcal{B}_{\rm c}}\right)^{\gamma-1}\cdot N_{\rm meas}\,. \tag{10}$$

Here the factor $\mathcal{Q}$ is determined by the fraction of the magnetic stars among all the stars of the selected spectral class, the value of $\mathcal{R}$ is a correction factor for the stars with fields smaller than the threshold fields, which can not be detected with the current techniques of magnetic fields measurement, and $N_{\rm meas}$ is a number of stars with measured magnetic fields.

Both $\mathcal{Q}$ and $\mathcal{R}$ values are poorly known. Let us suppose that the value of $\mathcal{Q}\approx 1/2$, and adopt a total fraction of magnetic B and A stars as 5 % (see, for example, Monin et al., 2002 and Shorlin et al., 2002). It means that the parameter $\mathcal{R}\approx 10\,\%$. Substituting this value to the relation (10), and adopting $\mathcal{B}_{\rm tr} = 0.4\,$kG one finds that $\mathcal{B}_{\rm c} = 25\,$G for the B stars and $\mathcal{B}_{\rm c} = 38\,$G for the A stars. If we adopt the parameter $\mathcal{Q}\approx 1$, than $\mathcal{B}_{\rm c} = 13\,$G and $\mathcal{B}_{\rm c} = 23\,$G for the B and A stars accordingly. It is important to note that the values of $\mathcal{B}_{\rm c}$ we obtained are close to the minimal values of $\mathcal{B}$, detected for the BA stars.

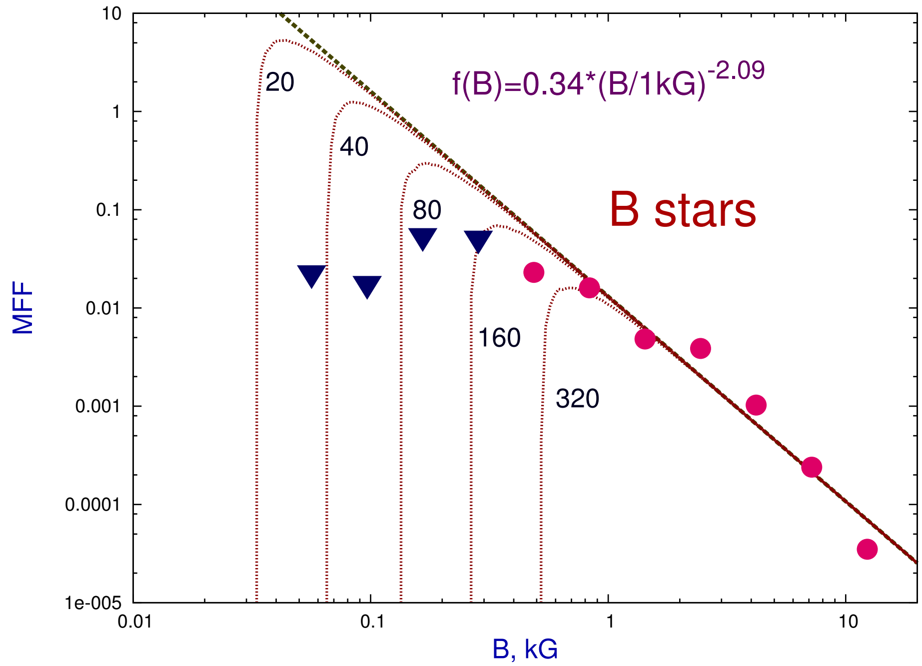

Figure 6: The magnetic field function for the B stars (points and triangles). The magnetic field function power law approximation (6) is marked by the thick dashed line. The corrected magnetic field functions are shown with dotted lines marked with values of $\sigma_{\mathcal{B}}$.

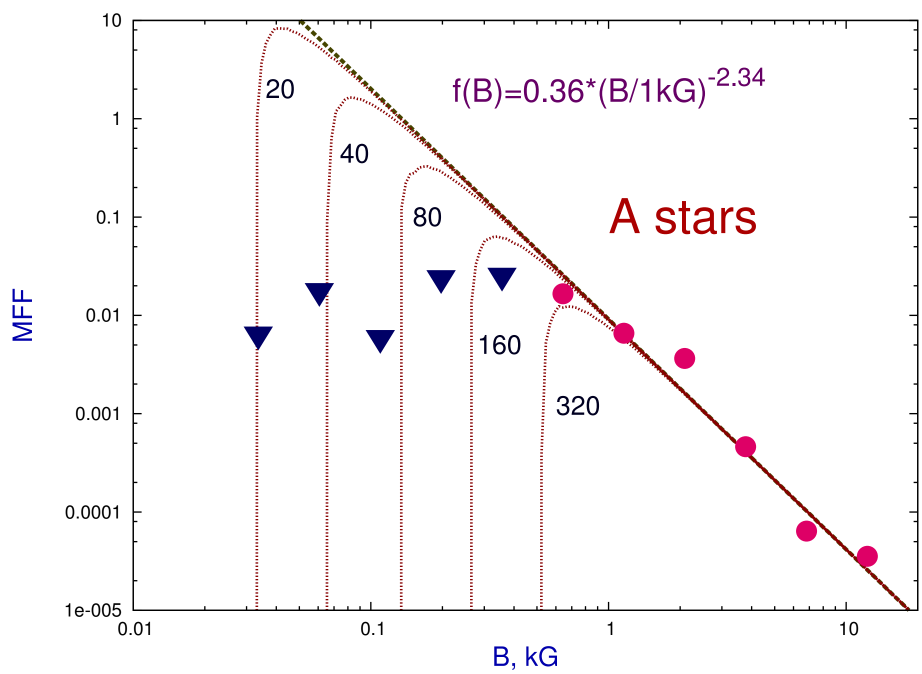

Figure 7: The same as in Fig. 6 but for the A stars

Using the relation (10) it is possible to write the corrected magnetic field function for the observational bias as

$$f(\mathcal{B})^{\rm corr} = \begin{cases} \frac{P(\sigma_{\mathcal{B}},\mathcal{B})}{\mathcal{QR}} A_0 \mathcal{B}_{\rm c}^{-\gamma} & ,\mathcal{B} < \mathcal{B}_{\rm c}\,, \\ \frac{P(\sigma_{\mathcal{B}},\mathcal{B})}{\mathcal{QR}} A_0 \mathcal{B}^{-\gamma} & ,\mathcal{B} \geq \mathcal{B}_{\rm c}\,. \end{cases} \tag{11}$$

In Fig. 6 we present the corrected magnetic field function for various values of the parameters $\sigma_{\mathcal{B}}$ for the B stars. The same but for the A stars is given in Fig. 7. In contrast to the procedure of the magnetic field normalization described by Kholtygin et al. (2010a, 2010b), we have normalized the magnetic field functions on the total number of stars of the selected spectral class, whereas in the cited papers we used the normalization over the number of stars with measured magnetic fields for the given value of $\sigma_{\mathcal{B}}$.

4 Magnetic Fluxes

4.1 Main Relations

The total magnetic flux $\mathcal{F}$ at the level of stellar photosphere in the spherical coordinate system (θ, φ) can be found as

$$\mathcal{F} = \iint\limits_S (\boldsymbol{B} \cdot \boldsymbol{n})\, dS = R_*^2 \int\limits_{-\pi}^{\pi}\int\limits_0^{2\pi} B_r \sin\theta d\theta d\varphi\,. \tag{12}$$

Here $\boldsymbol{B}$ is the magnetic induction vector, B_r is its radial projection, $\boldsymbol{n}$ is the normal to the stellar surface and R_* is the stellar radius. For the case of a dipole field from (12) we have:

$$\mathcal{F}_d = (4/3)\pi B_p R_*^2\,. \tag{13}$$

To estimate the magnetic fluxes in stars with known values of $\mathcal{B}$, we shall use the following relation:

$$\mathcal{F} = 4\pi\mathcal{B}R_*^2\,, \tag{14}$$

This gives a good estimation of the magnetic flux even in the extreme case of the dipole field $\mathcal{F} \approx 5/3\mathcal{F}_d$. For more complex field configurations, the difference between the exact value of $\mathcal{F}$, calculated using (12) and (14) is smaller.

We calculate the magnetic fluxes of all the stars presented in (Bychkov et al., 2003) and (Bychkov et al., 2009) catalogues. For the evaluation of the stellar radii we used the radii taken from the CADARS catalogue (Pasinetti Fracassini et al., 2001). For the stars with unknown radii we used the standard mass–luminosity relation from (Cox, 2000).

Using (14) we calculate the magnetic fluxes for all the stars with measured magnetic fields. The dependence of stellar magnetic fluxes on the stellar radii is presented in Fig. 8. The magnetic fluxes are located mainly in the interval $\mathcal{F} \in [10^{25}, 10^{27}]$, as it is visible in Fig. 8.

We also note that the majority of stars with measured magnetic fields are located in the interval of 2 to 3 $R_\odot$. This is the region of radii of the Ap and late Bp stars. The mean magnetic fluxes of stars appear to be in a narrow interval, $25.5 < \log\overline{\mathcal{F}} < 26.5$.

4.2 Magnetic Flux Distribution

The differential distribution function of magnetic fluxes $f(\mathcal{F})$ can be determined, as it was made by Kholtygin et al. (2010b), via the following relation:

$$N(\mathcal{F}, \mathcal{F}+\Delta\mathcal{F}) \approx N f(\mathcal{F})\Delta\mathcal{F}\,, \tag{15}$$

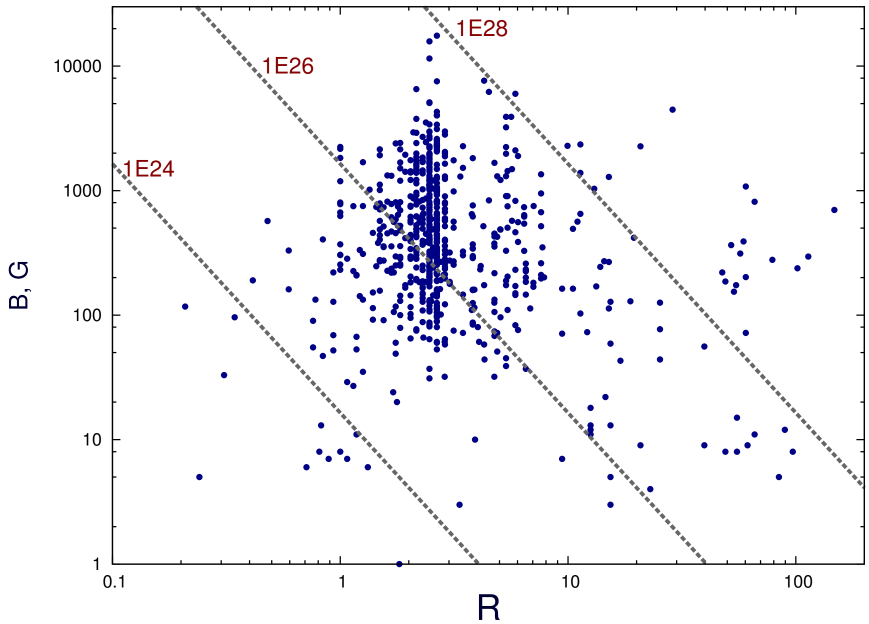

Figure 8: Stellar magnetic fluxes of normal stars vs. their radii in the solar radius. Positions of normal stars in the catalogue (Bychkov et al., 2009) are marked by points. The dashed lines show the lines of the constant magnetic flux: 10^{24}, 10^{26} and 10^{28} G·cm^2 (from bottom to top).

where $N(\mathcal{F},\mathcal{F}+\Delta\mathcal{F})$ is the number of stars with fluxes in the interval $(\mathcal{F},\mathcal{F}+\Delta\mathcal{F})$, and N is the total number of stars with measured $\mathcal{F}$.

Our study shows that the distribution of stellar magnetic fluxes is often asymmetric due to the missing stars with small magnetic fluxes. In Fig. 9 we plot the distribution of the logarithm of magnetic fluxes for the B stars.

The magnetic flux distribution of the A stars is given in Fig. 10. As it can be seen from Figs.9 and 10, the distribution of the magnetic flux logarithm of the B and A stars can be approximately described by a normal law. It means that the distribution of the magnetic fluxes obeys the log–normal law.

5 Discussion of Results

The analysis of Figs.6 and 7 leads us to the conclusion that although the function $f(\mathcal{B})$ for $\mathcal{B} > 0.40$ kG, derived from the magnetic field observations can be described by the reconstructed function $f^{\rm corr}(\mathcal{B})$, the magnetic field function in the range of weak magnetic fields cannot be reproduced at any $\sigma_{\mathcal{B}}$. The explanation of the cutoff in the magnetic field function for $\mathcal{B} < 0.40$ kG only by the observational selection effects may be incomplete since the present–day field measurement accuracy is about of $20-40$ G (Auriere et al., 2007; McSwain, 2008; Schnerr et al., 2008). It means that the magnetic fields in the range $60-120$ G may be detected without great problems. One can see from Figs.6, 7 that a number of stars with magnetic fields in this range may be detected.

Auriere et al. (2007) presented magnetic field measurements of 28 Ap/Bp stars. For 24 stars they fitted the phase dependence of the measured longitudinal field B_l in the model of an oblique rotating dipole, and obtained the polar field strengths B_p. A histogram for the stars in bins of $\Delta \log B_p = 0.2$ dex was constructed from the calculated values of B_p. The number of stars with $B_p < B_p^{\rm th}$, where $B_p^{\rm th} = 0.30$ kG is a threshold value of magnetic field have been found to be very

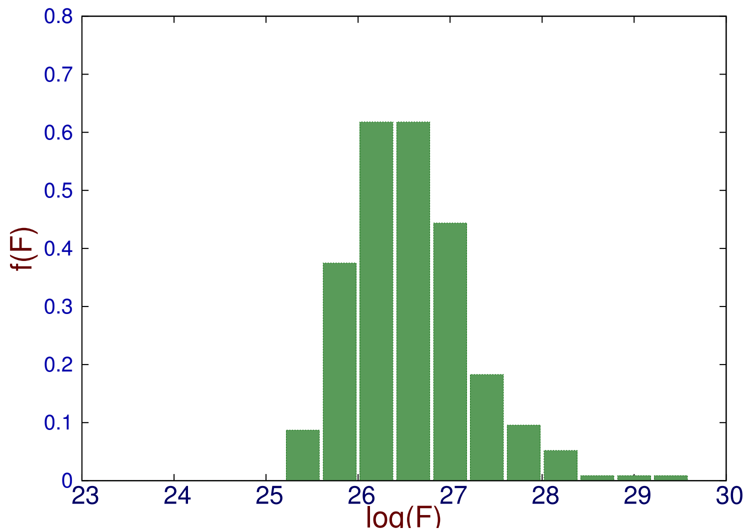

Figure 9: The distribution function of the logarithm of magnetic fluxes $\mathcal{F}$ for the B stars

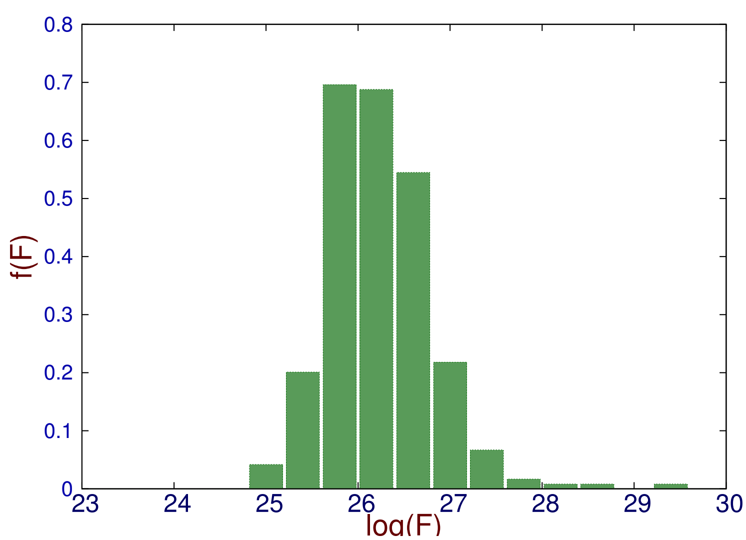

Figure 10: The same as in Fig. 9, but for the A stars

small. The conclusion is supported by the results reported by Wade et al. (2009) on the statistical study of dipole field strengths of the Ap stars within 100 pc from the Sun. Their histogram of dipole field strengths for 31 Ap stars also shows an absence of stars with $B_p < 0.30\,\mathrm{kG}$.

Monin et al. (2002) pointed out that the magnetic field function does not increase monotonically up to small field strengths. Instead, the slope of the magnetic field function in the region of the surface fields $B_s = 1-5\,\mathrm{kG}$ is significantly smaller than that in the region of larger fields. Wade et al. (2009) argued that there appears a plateau in the magnetic field distribution function around $B_p \approx 1\,\mathrm{kG}$.

If we use the relation

$$\mathcal{B} \approx 0.19\, B_p\,,$$

obtained by Kholtygin et al. (2010a), we can conclude that the threshold value $B_p^{\rm th} = 0.30\,\mathrm{kG}$ corresponds to a critical value to the *rms* mean magnetic field $\mathcal{B}^{\rm crit} = 57\,\mathrm{G}$. This value is close to the minimal value of the *rms* mean magnetic field $\mathcal{B} = 40-60\,\mathrm{G}$ for which the magnetic field function can be determined (see Figs.6, 7). The region of the small slope or the plateau of the magnetic field function by Monin et al. (2002) corresponds to values of the *rms* mean magnetic field $0.2\,\mathrm{kG} \leq \mathcal{B} \leq 1\,\mathrm{kG}$ that is close to our field range with a very small slope in the magnetic field function.

Firstly, Glagolevskij & Chuntonov (2000) proposed an explanation of a relatively small number of stars with measured magnetic fields in the range of$\mathcal{B} = 0.20-0.40\,\mathrm{kG}$. They suggested that if the mean stellar magnetic field is below some threshold value of $\mathcal{B}^{\rm th}$, then the field strength in the stellar atmosphere decreases almost to zero in a short time due to the processes of meridional circulation.

Recently, Auriere et al. (2007) suggested that some critical field strength does exist (corresponding to the observed one) above which stable magnetic field configurations can exist. In contrast, below this critical field strength any large–scale field configuration is destroyed due to instability. This instability generates opposite polarities at small–length scales, thus strongly reducing the magnitude of the longitudinal field due to the effects of cancellation and accelerating the Ohmic decay. For a sample of stars having both stable and unstable field configurations, this scenario would imply a strong jump in the measured values of the longitudinal field depending on the detection limit.

When a magnetic field is sufficiently weak to be changed by the differential rotation, the resulting field can be subject to various instabilities. As it is pointed out by Spruit (1999), the most vigorous of these instabilities is a pinch–type instability first considered in the stellar context by Tayler (1973).

Auriere et al. (2007) have estimated the critical magnetic field $B^{\rm crit}$ below which the winding–up process induces an instability of the large–scale magnetic field. The value of $B^{\rm crit}$ can be expressed in terms of the equipartition field $B_{\rm eq}$:

$$\frac{B^{\rm crit}}{B_{\rm eq}} = 2\left(\frac{P_{\rm rot}}{5\,\rm day}\right)^{-1}\left(\frac{R_*}{3\,R_\odot}\right)\left(\frac{T_{\rm eff}}{10^4\,K}\right)^{-1/2}\,. \tag{16}$$

Here $P_{\rm rot}$ is the stellar rotation period, R_* is its radius, and $T_{\rm eff}$ is its effective temperature. The equipartition field $B_{\rm eq}$ is determined from the condition of equality of gas and magnetic pressure. The value of $B_{\rm eq}$ is equal to $\approx 170\,\mathrm{G}$ at the surface of a typical Ap star with $\log g = 4$ and $T_{\rm eff} = 10^4\,\mathrm{K}$. Then the value of the critical magnetic field $B_{\rm c}$ is close to the observed 300 G threshold.

The relation (16) was obtained for the Ap stars, but it can also be used for other types of stars with large–scale regular magnetic fields. If we estimate the critical magnetic field value $B^{\rm crit}$ for the B and O stars, substituting the appropriate parameter values in the formula (16), we obtain the value $B^{\rm crit}$ of about $1-2\,\mathrm{kG}$ for the typical rotation periods of stars as $P_{rot} = 2-5$ days. Such relatively large values of $B^{\rm crit}$ may explain the small fraction of magnetic stars among the B and especially O stars. Only for slowly rotating OB stars, such as the O6pe star ϑ^1 Ori C with a long rotation period $P_{rot} = 15.4^d$, the value of $B^{\rm crit}$ will be $\sim 300\,\mathrm{G}$.

The above described scenario can qualitatively explain the existence of a lower limit in the magnetic field strength of the A stars, and especially OB stars, and also explain why the magnetic fields are observed only in a small fraction of OBA stars. If the initial magnetic field strength distribution of the intermediate-mass stars strongly increases towards the weak fields, a large majority of stars after the field formation would have weaker fields than the critical ones described by (16). The fields of such stars would be unstable and decay. And they would therefore show no magnetic fields on the main sequence.

A large scatter of stellar magnetic fields and their variation with the stellar age are probably the reasons of a large spread in magnetic fluxes in the main sequence stars of the same spectral subtype with similar radii, as one can see in Fig. 8. The mean effective magnetic field $\mathcal{B}$ was found to decrease with increasing τ (Hubrig at al., 2007), where τ is a relative age of the star on the main sequence. The decrease of magnetic field strength $\mathcal{B}$ and magnetic flux $\mathcal{F}$ in the A and B type stars on the main sequence with increasing τ was established by Kochukhov & Bagnulo (2006) and Landstreet et al. (2008).

To clarify this question, consider a sample of B4 − B9 stars with similar masses of (3-5) $M_\odot$, for which their absolute and relative main sequence lifetimes were determined by Kochukhov & Bagnulo (2006). The dependence of mean magnetic fluxes $\mathcal{F}$ for the stars of this sample on their age was analyzed by Kholtygin et al. (2010b). They found that $\mathcal{F}(\tau)$ could be fitted by an exponential function $\mathcal{F}(\tau) = \mathcal{F}_0 \exp(-\zeta\tau)$, where $\mathcal{F}_0 = 1.09 \cdot 10^{27}\,\mathrm{G{\cdot}cm^2}$ and $\zeta = 2.04$. A large rate of magnetic flux decay in the B stars can be explained mainly by the variation of their magnetic field since the variations in the radii of stars during their MS evolution are small. We may suppose that a similar dependence of the *rms* magnetic field on age holds both for the O and A stars.

6 Conclusion

During more than six decades starting with the discovery of stellar magnetic fields by Babcock (1947), the magnetic fields have been measured for more than a thousand stars of various spectral types. Based on the statistical analysis of the catalogues of magnetic fields by Bychkov et al. (2009), Romanyuk & Kudryavtsev (2008) and recent measurements, we analyzed statistical properties of the *rms* mean stellar magnetic fields $\mathcal{B}$ and magnetic fluxes $\mathcal{F}$. The following conclusions can be formulated from our analysis of statistical properties of magnetic fields and fluxes of the OBA stars:

- We can make a conclusion about a possible significant increase in the magnetic fields of B stars averaged over spectral subtypes in a comparison with the magnetic fields of O stars.
- The mean magnetic field distribution of the B and A stars $f(\mathcal{B})$ is a power law dependent on $\mathcal{B}$ with a power index $\gamma \approx 2.1-2.3$. The change in the slope of the function $f(\mathcal{B})$ almost up to zero for $\mathcal{B} < 400\,\mathrm{G}$ is detected.
- There exists a sharp decrease of mean magnetic fields of the BA stars below the critical value of $\mathcal{B}_c \approx 60\,\mathrm{G}$. This effect may be related to a dissipation of weak stellar surface magnetic fields.
- The magnetic flux distributions of the B and A stars are similar, and the logarithm of mean magnetic fluxes is $\log \overline{\mathcal{F}} \approx 26$.

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