Bull. Spec. Astrophys. Obs., 1998, 45, 84-92

Magnetic field function of white dwarfs

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Received April 16, 1998; accepted April 28, 1998.

Abstract. The frequency of magnetic white dwarfs is shown to decrease sharply with distance. This is caused by two reasons: 1) the observational selection — we observe cool stars only at short distances; 2) evolution of white dwarf magnetic fields (Valyavin and Fabrika, 1998) — the white dwarfs' average magnetic field increases with their cooling. We have estimated the real frequencies of hot (T > 10000 K) and cool (T < 10000 K) magnetic degenerates. The frequency of hot stars is $3.5 \pm 0.5\,\%$, their numbers are selection–independent to distances 80 ± 10 pc. The frequency of cool stars, as estimated for distances to 25 pc, is $\geq 20{\pm}5\,\%$. Nevertheless, we probably underestimate the number of cool magnetic degenerates among the coolest stars. A magnetic field function (MFF) of white dwarfs was studied in the range of surface magnetic fields from 6 kG to 1 GG. This function is a power function with a spectral index $\alpha = -1.5 \pm 0.1$. The MFF of hot degenerates (initial) and the MFF of cool degenerates (current) are discussed. The slopes of both functions are the same in spite of the strong magnetic field evolution in white dwarfs. This confirms the idea that the magnetic field evolution in degenerates does not depend in a first approximation on the initial field strength. We have concluded from the MFFs analysis that the probable minimal large–scale magnetic field strength of hot white dwarfs is about $B_s \approx 1 - 10$ kG, and this value in cool white dwarfs is about $B_s \approx 10 - 50$ kG.

Key words: stars: white dwarfs: magnetic fields: evolution

1. Introduction

The frequency of white dwarfs depending on their magnetic fields was first discussed by Angel et al. (1981). They estimated the average frequency of magnetic white dwarfs to be of about 1–2 %. They concluded that this value is a lower limit since the number of magnetic stars is underestimated because of observational selections. They suggested the real fraction of magnetic white dwarfs to be about 5 %. Over the past 15 years the number of spectroscopically classified degenerates increased by more than a factor of 4. Approximately the same factor holds for the known magnetic white dwarfs. Observational data available today enable a closer examination of the distribution of degenerate frequencies over magnetic fields to be performed. We call this distribution a magnetic field function (MFF) — the probability density of occurrence of stars as a function of their surface magnetic field strength. The MFF is defined in exactly the same way as the well-known mass function. It is very important that the MFF is normalized. This means that it is possible to study stellar magnetism in the non-observed ("weakly magnetized") region through fitting the MFF by a particular law and extrapolation.

The available observational data on white dwarfs allow the MFF for these stars to be derived in an interval of a few decades of magnetic field strength. The observed distribution of degenerates over magnetic fields was also studied by Schmidt and Smith (1995) and Putney (1977). In our previous papers (Fabrika and Valyavin, 1997; Fabrika et al., 1997; 1998) it was shown that the MFF of white dwarfs can be fitted in a first approximation with a single power relation. The MFF is consistent with the assumption that magnetic white dwarfs are not a particular class of stars, but represent the most magnetized stars, i.e. they are a part of common distribution of degenerates over magnetic fields. It was also shown, when examining the MFF, that it is of great importance to take into account observational selections and the techniques of magnetic white dwarfs detecting.

Herein we analyse the current (or observed) MFF of degenerates on the basis of all published observational data. Both frequency of magnetic white dwarfs and the mean strength of their surface magnetic fields are strongly dependent on temperature and age of these stars (Valyavin and Fabrika, 1998). Obviously we have to regard here for the magnetic field evolution. That is why we evaluate also the MFF for about zero–age degenerates too — the initial MFF. Knowledge of these functions, both current and initial, will make possible a some progress in the problem of magnetic fields evolution, and allow to estimate possible minimum surface magnetic field strengths of white dwarfs.

Comparison of the MFF of degenerate stars with the same function of the main sequence stars may help solve the problems of relationship between magnetism of main sequence stars and that of white dwarfs, of magnetic field evolution in stars. Angel et al. (1981) were in fact the first to substantiate the assumption that magnetic white dwarfs may be formed from magnetic Ap and Bp stars — magnetic and chemically peculiar stars. The spatial density – lifetime ratio they found proved to be the same for magnetic white dwarfs and magnetic stars. This hypothesis accounts well for the principal observational appearances (Schmidt and Smith, 1995; Putney, 1997). It seems the most attractive. It is likely that the magnetic fields of $B \gg 1$ MG are impossible to produce in white dwarfs by dynamo mechanisms. This suggests that the magnetic fields of degenerate stars are relic. In this light a study of the MFF of main sequence stars is of importance. A preliminary analysis of the MFF of main sequence stars (Bychkov et al., 1997) has shown that this can also be represented by a simple power relation.

2. Observational data, analysis of observational selections

Determine the magnetic field function as $P_B(B_s) = \Delta P(B_s)/\Delta B_s$, where $\Delta P(B_s)$ is a probability of finding a white dwarf with a surface magnetic field in the interval $B_s, B_s + \Delta B_s$. $\Delta P(B_s)$ can be evaluated as a ratio of known magnetic degenerates number with magnetic field strengths from the interval $B_s, B_s + \Delta B_s$ to the total number of white dwarfs observed with an accuracy which allows a magnetic field to be detected in this interval. By direct counting of magnetic white dwarfs in different intervals of fields one can estimate the frequencies of magnetic stars, that is to derive the MFF. From all the published data we have composed a sample of 53 magnetic white dwarfs with surface magnetic fields from a few hundred kG to about 1 GG (Table 1). The first column of the table is object's name, the second — stellar magnitude, the third — effective temperature, the fourth — surface magnetic field value in MG, the fifth and the sixth columns provide masses in $M_\odot$ and ages in Gy, estimated by Valyavin and Fabrika (1998); in the last column we present the references to the magnetic field measurements (and partially, temperatures, see Valyavin and Fabrika, 1998). The references are given in the notes under the table. This sample contains white dwarfs whose magnetic fields have been measured directly from spectra, specropolarimetry and polarization measurements in broad bands.

Nearly all classified white dwarfs (about 2100 stars) including magnetics from Table 1 are presented in the catalogue by McCook and Sion (1999). Magnetic measurements and the ways the magnetic white dwarfs were detected are influenced by observational selections. So the original data available from observations could not be considered as random.

In order to derive $\Delta P(B_s)$, one has to know selection coefficients (Valyavin and Fabrika, 1998), or to pick out such samples of magnetic and nonmagnetic white dwarfs which would have about equal their selection coefficients. Below at the analysis of magnetic white dwarfs frequencies in different field intervals we consider different selections such as serendipitous discoveries, star's rotation. Here we will focus on the main and general selection — magnetic white dwarfs frequencies dependence on star's brightness, i.e. on their temperature and distance.

Even the faintest magnetic white dwarfs are located at distances no greater than 500–600 pc. Assuming the radial velocity dispersion of white dwarfs to be $\gtrsim 10^6$ cm/s and an age $\sim 10^9$ years, find that the region of mixing of any possible original space inhomogeneities of such stars is considerably larger than 600 pc. Thus the main criterion of the absence of the selection must be the fact that the frequency of magnetic white dwarfs is independent on distance.

The nearest and brightest stars are well studied and therefore a frequency of magnetic white dwarfs among them is least selection–dependent. Beginning with some distance D the selection of magnetic white dwarfs detection appears. Since the star brightness is also determined by temperature, the distance D must be dependent on effective temperature of degenerates.

Fig. 1 displays how the frequency of magnetic white dwarfs depends on distance. The frequency P_m was found for all the stars having known parallaxes, but the magnetic white dwarfs only with surface magnetic fields over 1 MG were considered. The horizontal bars show the distance bins. The best fit as a simple relationship $P_m(d) = C/(1 + d/D)$ is shown. The constant C here is a selectionless estimate of magnetic white dwarfs frequency; the constant D is a characteristic distance, at $d \gtrsim D$ the observational selection becomes significant. The best fit yields $C = 0.17 \pm 0.03$ and $D = 25 \pm 5$ pc.

A very important conclusion concerning real frequency of magnetic white dwarfs follows from Fig. 1. The conventional frequency (see e.g. Angel et al., 1981; Jordan, 1997) of magnetic degenerates is 2–4 %. Indeed this is, if one considers all the data with no allowance made for observational selections. From the total number of the known degenerates the incidence of magnetism is $53/2100 \approx 0.025$. For instance, from the white dwarfs located in the interval of distances from 25 to 100 pc, i.e. from hot stars alone, the fre-

Table 1: *List of magnetic white dwarfs*

Name	V	$T_{\rm eff}$, K	B_s, MG	M, $M_\odot$	t, Gy	Ref
LHS 1038	14.36	6400	~ 0.1	0.72	3.58	29
LP 907-037	14.6	9500	~ 0.1	0.98	1.24	29
G234-4	16.38	4500	~ 0.12	0.75	7.07	34,35,38
LHS1415	15.8	6000	0.12			38
GD 077	14.80	10000	~ 0.9	0.75	0.5	29
PG 1220+234	15.62	27200	~ 2	0.62	0.07	29,35
G141-2	15.88	6000	2	0.75	1.43	2,35
PG1658+440	15.02	30500	2.2	1.28	0.16	1,35,37
LB8827	15.8	2000	~ 3	0.83	0.126	34,35,38
KUV08165+3741	15.6		4			26
MWD0159-032	17.10	26000	4-5		0.068	3,35
G62-46	17.11	6050	5	0.72	2.9	29,31,35
LHS 1734	15.97	5300	5-6	0.59	3.02	30,29,35
HS1254+3440	17.00	12500	6-7		0.63	9,29,35
LHS 2273	16.54	6000	6-7	0.39	1.76	29,35
MWD0307-428	16.30	25000	7-8		0.082	3,35
GD90	15.74	11000	7-8	0.86	0.98	4,35
PG 1312+098	16.4	15000	7-8	0.73	0.2	6,29,35
G99-37	14.58	6300	~ 8	0.84	5.87	5,29,35
KUV03292+0035	16.70	19000	8			7
HS1440+7518	14.9	40000	8		0.01	29,35
KUV16032+1735	15.6		8		0.082	25
G 183-35	16.92	7000	~ 10	0.83	3.6	29,32,35,38
GD356	15.04	7500	8-11	0.85	2.8	8,29,35
LHS 1044	15.33	6000	11-12	0.8	5.82	29,30,35
KPD0253+5052	15.22	15000	12-13		0.16	6,29,35
RE 0616-649	18.40	35000	13-14		0.01	29,35
G256-7	15.99	5600	~ 15	0.83	8.05	32,35,38
G99-47	14.11	5600	14-18	0.63	4.04	11,29,35
LBQS1136-0132	18.00	15000	17-18		0.37	10,29,35
HE 1045-0908	16.40	9000	20-21		1.419	29,35
PG 1533-057	15.33	17000	21-24	0.49	0.06	12,29,35
Feige 7	14.50	20000	23-25	0.78	0.15	13,29,35
BPM 25114	15.74	20000	24-26	1.34	1.44	14,29,35
KUV2316+123	15.58	11800	30-40		0.74	6,29,35,37
GD116	15.96	16000	44-47	0.89	0.43	15,29,35
HE 1211-1707	16.9	22500	~ 50			33
ESO 439-162	18.8	5400	67	1.13	8.13	17,35
G195-19	13.86	8000	~ 70	0.98	4.6	18,29,35
HE0000-3430	15.0	7000	70	0.92	5.7	33,35
PG1015+015	16.3	14000	80-90	1.03	1.341	19,29,35
G227-35	15.05	7000	90-150	1.14	7.62	20,29,35
LP790-29	16.0	7500	~ 150	1.34	8.7	21,29,35
G240-72	14.15	7500	~ 150	1.15	8.48	22,29,35
HE 0127-311	16.1	18000	~ 200		0.19	29,33,35
GW+70 8247	13.20	15000	220-240	1.13	1.747	23,29,35
HE2201-2250	16.2	18000	200-250		0.19	33,35
RE 0317-853	16.00	50000	200-500	1.35	0.01	29,40
PG0945+245	14.3	16000	400-500	1.18	0.71	28,29,35,36
SBS1349+5434	17.6	11000	~ 500		0.87	27,29,35
G111-49	16.28	8400	~ 600	0.77	1.47	32,35,38
GD229	14.85	23000	$\sim$ 300-700	1.28	0.49	16,29,35,39
PG1031+234	15.8	25000	300-700	1.11	0.047	24,29,35

Notes to Table 1:
(1) Liebert et al., 1983; (2) Greenstein, 1986; (3) Achilleos and Remillard, 1991; (4) Angel et al., 1974a; (5) Angel and Landstreet, 1974; (6) Schmidt and Norsworthy, 1991; (7) Wegner and Boley, 1993; (8) Greenstein and McCarthy, 1985; (9) Hagen et al., 1987; (10) Foltz et al., 1989; (11) O'Donoqhue, 1980; (12) Liebert et al., 1985; (13) Liebert et al., 1977; (14) Wickramasinghe and Bessel, 1976; (15) Saffer et al., 1989; (16) Schmidt et al., 1990; (17) Ruiz and Maza, 1989; (18) Angel et al., 1972; (19) Angel, 1978; (20) Cohen et al., 1993; (21) Liebert and Strittmatter, 1977; (22) Angel et al., 1974b; (23) Angel et al., 1985; (24) Schmidt et al., 1986; (25) Wegner and Swanson, 1990; (26) Wegner and Boley, 1993; (27) Liebert et al., 1994; (28) Liebert et al., 1993; (29) Schmidt and Smith, 1995; (30) Bergeron et al., 1992; (31) Bergeron et al., 1993; (32) Putney, 1995; (33) Reimers et al., 1996; (34) Putney, 1996; (35) Jordan, 1997; (36) Glenn et al., 1994; (37) Friedrich et al., 1996; (38) Putney, 1997; (39) Jordan et al., 1998; (40) Barstow et al., 1995.

quency is $P_m = 0.029 \pm 0.08$. It is obviously that the true frequency can be found from the nearest stars only. From Fig. 1 this equals 0.17 ± 0.03. Cool stars are the main contributors to this value. We cannot leave out of account the evolution of magnetic fields, that is why it is necessary to find the frequencies of hot and cool magnetic white dwarfs separately.

Valyavin and Fabrika (1998) have shown the magnetic fields of degenerates to evolve and the frequency to depend strongly on effective temperature. With the stars cooling the frequency of magnetics among them enhances. It is apparent that cool (faint) degenerates are observed only in the closest vicinity of the Sun. The observed sharp drop in the frequency of magnetic white dwarfs at distances $d \gtrsim 25$ pc is accounted for by the fact that we lose cool degenerates. This can be well seen in Fig. 2, where we show the frequency of hot (both magnetic and non-magnetic) degenerates P_h with temperatures over 10000 K (filled circles) and that of cool stars P_c with temperatures from 4000 to 10000 K (open circles). We used all white dwarfs for which it was possible to find temperature and distance (1054 stars, Valyavin and Fabrika, 1998). The same drop in frequency of cool stars is seen there. This figure provides a prove of the assumption that the selection of magnetic white dwarfs caused by different distances is related to temperatures of stars. It also implies that in the effective temperature intervals examined the shares of cool and hot degenerates are 60 % and 40 %, respectively.

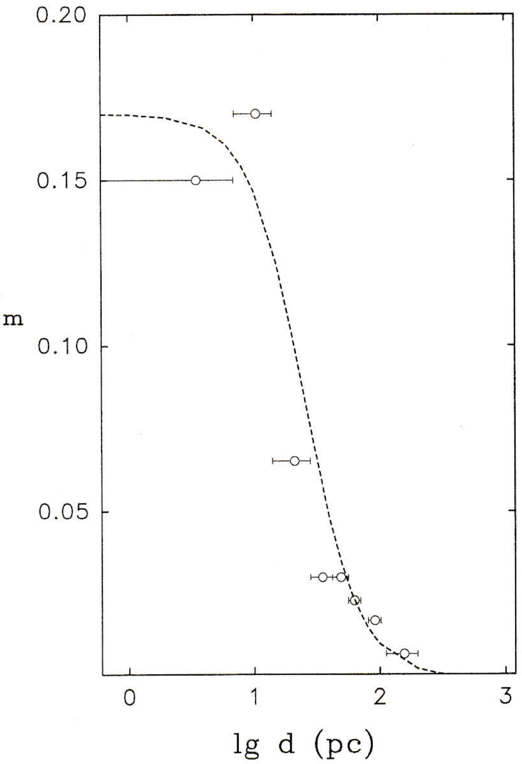

Figure 1: *The frequency of magnetic white dwarfs as a function of distance. Dashed line is the best fit with the relationship* $P_m(d) = C/(1 + d/D)$.

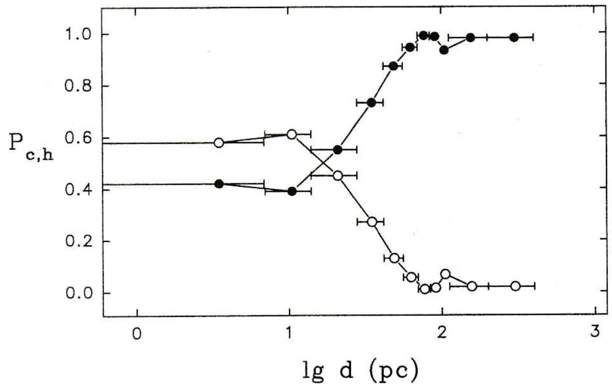

Figure 2: *The frequency of hot both magnetic and non-magnetic degenerates* P_h *with temperatures over 10000 K (filled circles) and that of cool stars* P_c *with temperatures from 4000 to 10000 K (open circles) as a function of distance.*

The frequency of magnetic white dwarfs versus distance in the two chosen temperature intervals is shown in Fig. 3 (filled circles for hot stars, open circles for cool stars). The best fit by the function $P_m(d) = C/(1 + d/D)$ for hot magnetic white dwarfs yields the constants $C = 0.035 \pm 0.005$ and $D = 80 \pm 10$ pc. Thus the sample of hot magnetic white dwarfs at the distances up to 80 pc may be considered to be more or less complete, and their frequency is estimated as 3.5 ± 0.5 %.

For cool white dwarfs it is not easy to distinguish a plateau in $P_m(d)$. In Fig. 3 we see the observational selection to be very strongly dependent on distance. The frequency obtained from the bin of nearest stars, which corresponds to 0–10 pc, is the most correct and selectionless estimate. Taking $D = 25$ pc, find that the best fit for cool stars yields $C = 0.20 \pm 5\,\%$. Thus magnetic white dwarfs account for $\gtrsim 20 \pm 5\,\%$ of cool stars. The coolest white dwarfs ($T \approx 4000$ K) are difficult to detect even in the closest vicinity of the Sun. That is why, we believe that the value we have found may be a lower limit of the frequency of cool magnetic white dwarfs in the temperature interval 4000–10000 K. The significant difference in frequencies of magnetic white dwarfs of different temperatures in Fig. 3 confirms the magnetic field evolution in white dwarfs Valyavin and Fabrika, 1998).

Now we can make a real estimate of the total frequency of magnetic white dwarfs, taking into account hot and cool stars, separately. Taking the proportions of hot and cool stars to be 40 % and 60 %, respectively (Fig. 2), find the total frequency of magnetic white dwarfs to be $\gtrsim 13.5 \pm 3\,\%$. The inequality here allows for possible shortage of the coolest stars. This estimate is of a formal character, since in the light of the observed strong evolution of magnetic fields, any interpretation of magnetism peculiarities in degenerates must take up the evolution, i.e. the dependence on temperature.

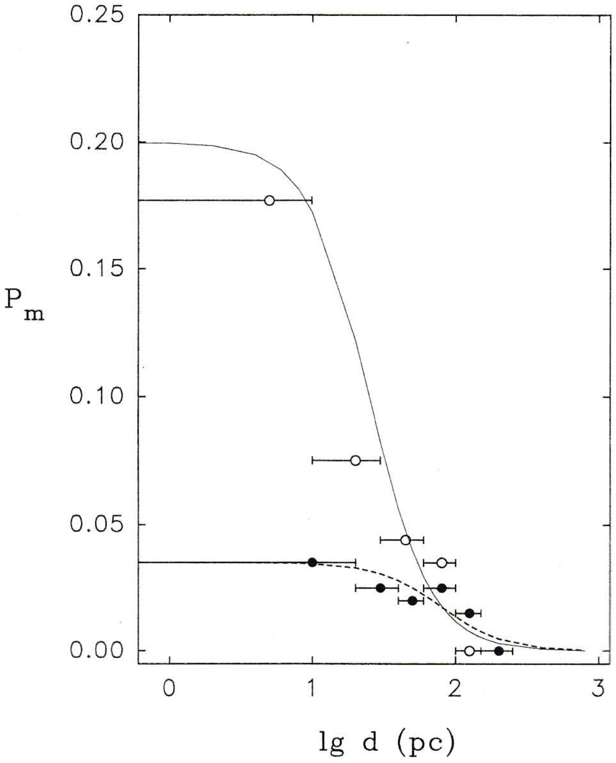

Figure 3: *The frequency of magnetic white dwarfs versus distance in the two chosen temperature intervals: filled circles for hot stars, open circles for cool stars. Dashed and solid lines are the best fits.*

3. Magnetic field function

Considering the magnetic field evolution we examine different versions of the MFF: for the whole sample of magnetic degenerates, for the nearest stars (i.e. the coolest), for hot stars and also for the most massive stars, as there was selected (Valyavin and Fabrika, 1998) a separate population of "ultramassive — ultramagnetic" degenerates. It is worth–while to speak about the initial (referred to about zero age) MFF. The MFF derived from hot (young) degenerates alone will be an approximation of the initial function. Now we will describe the procedures used to derive the MFF in different intervals of magnetic field strengths. The intervals are chosen in accordance with the techniques of observing magnetic white dwarfs, i.e. the methods used to discover these stars. It is convenient to derive the MFF in intervals where the magnetic field strength changes by one order of magnitude (decade intervals). That is why, if an interval determined by the observing technique is larger, we break it up into decade intervals.

Interval $10^6 < B_s \lesssim 10^9$ *G*

In this interval magnetic degenerates are detected by a simple visual examination of spectra owing to the large value of Zeeman splitting. Some magnetic white dwarfs were discovered by broad–band polarimetry (Angel et al., 1981) of degenerates with continuous spectra (DC). Since such measurements were made not for all DC stars classified by spectrum, a part of magnetic objects may be lost. The contribution of such lost stars is probably insignificant since the total share of DC stars among the known degenerates is small (no more than 10–15 %).

To estimate the MFF in this interval the following procedure is applied. Among all spectroscopically classified degenerates from the catalogue by McCook and Sion (1999), $N \approx 2100$ stars, we pick out N_m of white dwarfs with surface magnetic field lying inside one decade of field variation. The frequency of magnetic degenerates in the given interval is N_m/N. The probability density $P_B(B_s)$ is found by dividing the frequency by the field interval, its dimension is G^{-1}. This mean density estimate is referred to the middle of the interval being examined.

To reduce observational selections each known magnetic degenerate was considered by the way it had been discovered. We isolated only those magnetic white dwarfs which were detected as magnetic stars, when observers worked over lists of degenerates or degenerate candidates, i.e. when nonmagnetic stars were studied too. Magnetic white dwarfs detected serendipitously in some other way (for instance, from

X–ray radiation while observing other objects) were rejected. A total of 43 magnetic degenerates are involved in the MFF analysis in this magnetic field interval.

In the subinterval $10^6 < B_s < 10^7$ G (15 magnetic degenerates) the observational selection appears, connected with the fact that Zeeman line splitting is comparable with line widths. That is why, from spectra of low spectral resolution and signal/noise ratio a magnetic degenerate may be missed. We have already discussed this effect (Valyavin and Fabrika, 1998). It is important only at temperatures from 8000 to 25000 K, when Stark broadened hydrogen lines are strong. The effect is well visible in Fig. 3 in the paper by Valyavin and Fabrika (1998). It can readily be taken into account by introducing a selection coefficient. Indeed, the overall frequency of magnetic stars (of all temperatures) in this particular magnetic field interval is 1.4 times as low as the frequency of magnetic stars with fields $B_s > 10$ MG (the integrals over two low curves in Fig. 3 of the paper quoted are compared). It is this coefficient that we have to multiply the frequency in the interval of 1–10 MG by. Note that such a selection should be taken into account in this field strength interval only.

Interval $10^5 < B_s < 10^6$ *G*

These and weaker magnetic fields are impossible to detect directly in degenerate spectra since Zeeman splitting cannot be resolved against the background of broad lines. In present–day broad-band polarization measurements an accuracy of $< 0.3\,\%$ may be achieved, which is sufficient to detect magnetic fields of $B_s \lesssim 1$ MG. In fact, however, white dwarfs with magnetic fields $B_s < 1$ MG are discovered only by measuring the field longitudinal component — effective magnetic field — in special polarimetric spectral observations (Angel et al., 1981; Bychkov et al., 1991; Schmidt and Smith, 1995; Fabrika et al., 1997; Putney, 1997; Valyavin et al., 1997).

A new selection appears here which is connected with a star rotation. If in the central dipole model the angle j is that between the dipole axis and the line of sight and also the coefficient of limb darkening of star u = 1, then the following relationship takes place (Schmidt and Smith, 1995) $B_e \approx 0.4 \cdot B_p \cos j$, or $B_e \approx 0.56 \cdot B_s \cos j$, where B_e is the effective magnetic field, B_p — the pole field of the dipole, and B_s — the surface magnetic field. The angle j may take different values because of a random orientation of magnetic dipoles and also because of rotation. There is the probability of zero magnetic field detection in isolated observations. It is obvious that this probability is independent on rotation period because observation will always fall within a random phase of rotation. It is determined only by the dipole orientation, the accuracy of observations and the white dwarf magnetic field strength. When averaging over dipole orientation angles, the probability of null detection (Schmidt and Smith, 1995) is $p \approx 2.5 \cdot B_{lim}/B_p$, or $p \approx 1.79 \cdot B_{lim}/B_s$, where B_{lim} is an accuracy of magnetic field measurement, which is usually taken equal to 2σ. When this observational selection is taken into account, a frequency should be multiplied by $1/(1-p)$.

Here we consider all white dwarfs observed with analysis of circular polarization in hydrogen line wings (Bychkov et al., 1991; Schmidt and Smith, 1995; Putney, 1997; Fabrika et al., 1997), 193 stars altogether. Among them five stars have been discovered whose effective magnetic fields range from a few tens to a few hundred of kG. To find a surface magnetic field the well–known average statistical formula $B_s \approx 3B_e$ (Angel et al., 1981) can be employed. The surface fields of these four stars are > 100 kG, i.e. they fall within the interval under study (the stars LHS 1038, LP 907–037, G 234-4, LHS 1415 and GD 077 in Table 1). A typical accuracy $< \sigma >$ of magnetic field measurements in the list considered is 10 kG, i.e. $B_{lim} = 20$ kG.

The application of the approximation by Schmidt and Smith (1995) for the probability p in such a wide B_s interval as one decade seems unreasonable. This approximation may be made most correctly for each individual star. To estimate the probability of null detection we may be guided by these 5 available degenerates only, assuming that other stars are alike and may have been missed in the same way. So we obtained from these stars $< p > \approx 0.3$, which was taken into account. As previously, the density is estimated by division of the number of magnetic objects by the number of stars in the sample (193).

Interval $10^4 < B_s < 10^5$ *G*

Not a single object with detected magnetic field has been yet appeared in this interval. That is why, here one can estimate the upper limit of P_s. From the sample of 193 stars investigated by Zeeman spectroscopy, we choose stars whose measurement errors do not exceed the limiting $\sigma(B_e)$, which satisfies $2\sigma(B_e) \cdot 3 = 10$ kG, i.e. corresponding lower edge of the interval, $B_s = 10$ kG. We find this limiting error to be $\sigma(B_e) = 1.7$ kG. Altogether there are 6 degenerates that have been measured with an accuracy better than 1.7 kG. They are WD 1647+591, 1645+235 (Schmidt and Grauer, 1997); WD 0644+375 (Schmidt and Smith, 1995); 40 Eri B, WD 0713+584, 0232+035 (Valyavin et al., 1997). To find the upper limit note, that N observed stars did not show magnetic field in this particular field range, and suppose that N+1st star will show magnetic field in this range, then $1/(N+1)$ is the upper limit of the magnetic stars frequency. The selection probability is equal to 0.11 in this interval.

Interval $B_s < 10^4$ *G*

With an accuracy of $\sigma(B_e) \lesssim 1\,\mathrm{kG}$ magnetic fields were measured only in 3 degenerates WD 1647+591 (Schmidt and Grauer, 1997), 40 Eri B and WD 0713+584 (Valyavin et al., 1997). The accuracy obtained for WD 0713+584 from our continuous two hours row was about 1 kG. In two longest continuous observations of 40 Eri B formal upper limits of effective magnetic field were obtained: 0.5 ± 0.4 kG (1.3 hours of observations) and −0.1 ± 0.5 kG (3.6 hours of observations). In the last row the significant variations of magnetic field with a semiamplitude $B_e \approx 2.3 \pm 0.7$ kG, and a time–scale of about 2 hours were detected. This means that a surface magnetic field of this star may amount to about 7 kG. By the logic of a choice of magnetic field intervals when deriving the MFF, we can use these three stars for the frequency analysis only in the interval $6 \cdot 10^3 < B_s < 10^4$ G. In this interval the magnetic field was suspected in one of the three stars, thus the frequency is 1/3. If the magnetic field detection in 40 Eri B is not confirmed, this frequency will be an upper limit. Measurements of these stars were repeated many times, besides, individual measurements were sufficiently long. Therefore, the "missing" coefficient does not need to be introduced there.

4. Current magnetic field function

The observed MFF is shown in Fig. 4, the density of white dwarfs is shown versus the surface magnetic field strength. Filled triangle indicates the upper limit in the interval 10 kG $< B_s <$ 100 kG. Dots at a field strength over 1 MG are the estimates made from all known white dwarfs identified by spectra (McCook and Sion, 1999). Two left–hand dots are based on the data of Zeeman spectral observations. Statistical errors were computed by the Monte–Carlo technique. The MFF obtained does not contradict the idea that it can be fitted with a single simple relationship. The dashed line is a power approximation $P_B = AB_s^{\alpha}$, where the spectral index $\alpha = -1.5 \pm 0.1$, $A = 30 \pm 5\,G^{-1}$.

The MFF presented is actually derived from hot stars since there are no many cool degenerates both magnetic and non-magnetic in the total list (about 16 %). Three stars that represent the weakest magnetic point of the MFF are hot stars too. Thus the MFF derived from all the data may serve as an approximation of the initial MFF. Thanks to MFF normalization a minimum possible magnetic field may be estimated. By extrapolation of the MFF with the obtained spectral index to the region of weak magnetic field strengths, we find the minimum magnetic field to be 1 kG $< B_s(\mathrm{min}) <$ 10 kG in the given model.

Open triangles show the MFF derived using only the nearest stars located nearer than 25 pc. When examining the nearest stars, we have a sample of degenerates which is most free from observational selections. This sample comprises 14 stars, all of them, but one, are cool white dwarfs with T < 10000 K. It is evident that the MFF of cool stars is located upper on the plot than that of all stars as the frequency of cool magnetic degenerates is higher. The MFF shown by open triangles is a current MFF of cool white dwarfs.

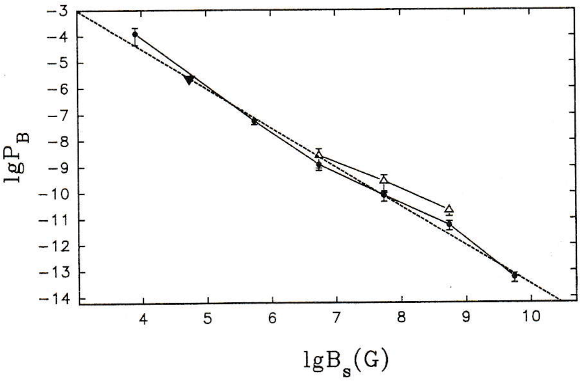

Figure 4: *The observed magnetic field function. Dots show the MFF obtained from all known magnetic white dwarfs (with* $B_s > 1$ *MG) identified from spectra. The filled triangle indicates the upper limit in the interval 10 kG* $< B_s <$ *100 kG. Two left-hand dots are based on the data of Zeeman spectral observations. Open triangles show the MFF derived using only the stars located nearer than 25 pc.*

It seems to us of great importance that the slope of the MFF of cool stars is the same as that of the MFF derived from all the data in despite of the essential evolution of magnetic fields. This may imply that the magnetic field evolution in degenerates does not depend, as it has been suggested by Valyavin and Fabrika (1998), on the initial magnetic field strength. Assuming that in a weak magnetic field range the behaviour of the cool stars' MFF is also described by the power with the index $\alpha = -1.5$, then we can find the constant $A \approx 105\,G^{-1}$. In turn, this implies that the minimum magnetic field in cool degenerates is 10 kG $< B_s(\mathrm{min}) <$ 50 kG.

The difference in the minimum surface magnetic field values of white dwarfs estimated in these two cases is well understandable. The average age of the 49 magnetic degenerates from which the first MFF version (of all data) has been derived is 2.2 ± 0.4 Gy. The average age of the nearest magnetic white dwarfs (inside 25 pc) is 5.5 ± 0.6 Gy. The observed magnetic fields rise with age (Valyavin and Fabrika, 1998), this accounts for the higher minimum magnetic field value in the nearest degenerates.

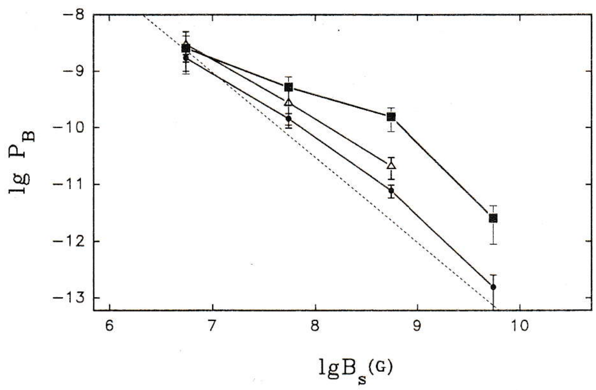

Figure 5: *The initial MFF (dots) derived from hot white dwarfs only with* $T_{eff} > 10000$ *K. The open triangles, the same as in Fig. 4, show the MFF derived from the nearest (cool) stars; the dotted line represents the power function* ($\alpha = -1.5$) *from the same figure. Squares show the MFF of massive white dwarfs with masses* $M > 1.1\,M_\odot$.

5. Initial magnetic field function

Consider the MFF derived from hot white dwarfs only with $T_{eff} > 10000$ K, whose average age is 0.5 Gy. To diminish observational selections, choose only the degenerates located at a distance no greater than 80 pc. Such a MFF will be quite a good approximation of the initial one. This MFF derived from 8 magnetic white dwarfs is displayed in Fig. 5 by dots. For comparison, the open triangles, the same as in Fig. 4, show the MFF derived from the nearest (cool) stars; the dotted line represents the power function ($\alpha = -1.5$) from the same figure.

The MFF slope of cool degenerates and that of hot stars are identical. We have already drawn this conclusion when comparing the MFF of cool stars with that derived from the whole sample of white dwarfs. We can now say with more assurance that the MFF slope is independent on white dwarfs' age. Indeed the mean ages of stars of these two groups (dots and triangles in Fig. 5) differ by more than an order. Their average magnetic fields and frequencies are also different. Nevertheless the slopes of the MFFs of these two groups are the same. We conclude from this fact that the evolution of magnetic fields of degenerates is in a first approximation independent on initial magnetic field.

In Fig. 5 we also present the MFF of massive white dwarfs with masses $M > 1.1\,M_\odot$ (squares). It was derived from 10 magnetic degenerates. This function is located above, because the frequency of massive white dwarfs among nonmagnetic stars (denominator in the expression of P_B) is about a factor of 9 less than that of massive white dwarfs among magnetic stars (Valyavin and Fabrika, 1998). The massive stars' MFF, however, has an essentially different slope. This is due to the fact that 7 out of 10 massive stars actually belong to the separate population of ultramassive–ultramagnetic degenerates (Valyavin and Fabrika, 1998). In that paper it was argued that magnetic white dwarfs are not a homogeneous class of objects, about 80 % of these stars are "ordinary" magnetic white dwarfs, whose average mass is $0.8\,M_\odot$ and $B_s < 100$ MG; about 20 % of these stars are ultramassive–ultramagnetic degenerates with an average mass of $1.15\,M_\odot$ and $B_s > 100$ MG. All the stars of the latter population fall within two most magnetic bins of the MFF in Figs. 4 and 5.

The aim of this paper does not involve an analysis of the ultramassive–ultramagnetic population. It is possible that the MFF of the latter population differs from the MFF of the main population of degenerates. It is important to have a more detailed study of this extremely interesting population of ultramassive–ultramagnetic white dwarfs.

6. Conclusion

We have found here that the frequency of magnetic white dwarfs increases sharply with decreasing distance to them. This is associated with the fact that we observe cool stars at small distances, so this confirms the evolution of magnetic fields in degenerates (Valyavin and Fabrika, 1998) — as these stars cool down their magnetic fields grow. Taking into account the observational selection, we have found real frequencies of cool and hot magnetic white dwarfs. The frequency of hot magnetic degenerates with the temperature > 10000 K is 3.5 ± 0.5 %; their sample may be considered selection–independent in a volume of 80 ± 10 pc in radius. The frequency of cool magnetic white dwarfs with the temperature < 10000 K is $\gtrsim 20 \pm 5$ %. The latter value has been obtained from nearest stars, $d < 25$ pc. In spite of the small distances considered we may lose some magnetic degenerates among the coolest stars.

In the surface magnetic field range from 6 kG to about 1 GG we have derived the magnetic field function of white dwarfs. It is fitted well with a power function with a spectral index $\alpha = -1.5 \pm 0.1$. From examination of this function we have estimated minimum possible strengths of the large–scale surface magnetic field of white dwarfs. The minimum field strengths in cool and hot degenerates are about 10–50 kG and 1–10 kG, respectively.

The initial and current (from hot and cool stars) MFF' slopes are the same in spite of strong observed evolution of magnetic fields. This supports the idea that the magnetic field evolution in white dwarfs does not depend in a first approximation on the initial magnetic field strength.

Acknowledgements. We are indebted to A. Putney, S. Jordan and G. Schmidt for the papers and preprints they made available to us and to G. McCook and E. Sion for the electronic version of the catalogue of white dwarfs. The work was supported by the RFBR grants N 95-02-03690, N 98-02-16554.

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