# Gravitation theory in multi-messenger astronomy I: comparison of geometrical and field approaches to the physics of gravitational interaction

Yu.V. Baryshev[1], S.A. Oschepkov[2]

[1]Astronomical Department, Saint Petersburg State University, Saint-Petersburg, Russia;
yubaryshevl@mail.ru

[2]Taurida Academy, V.I. Vernadsky Crimean Federal University, Simferopol, Russia

**Abstract** In modern theoretical physics there are two alternative possibilities of the description of gravitation: geometrical theory of the General Relativity Theory (GRT) of Einstein and non-metric theory of the material tensor Field Gravitation Theory (FGT) of Feynman. In this first report general provisions of these theories are stated: basic principles, Lagrangians, equations of the field and equations of the motion. These equations will be used for interpretations of observations in multimessenger astronomy, which discussed in our second report.

**Keywords:** relativistic astrophysics, gravitation, lagrangian formalism, quantum field theory.

## 1. Gravity physics as the bases of relativistic astrophysics.

The multi-messenger astronomy deals with the most energetic processes in the Universe such as compact relativistic objects (neutron and quark stars), candidates of black holes having stellar and galactic masses, gravitational radiation, massive supernova explosions, gamma ray bursts, jets from active galactic nuclei. Relativistic gravitational collapse is the source of the highest energy extraction from astrophysical objects and this is why the gravitation theory is the fundamental basis for interpretation of violent events in multi-messenger astronomy.

Since the beginning of the 20th century there are two really alternative approaches for the description of gravitational interaction in theoretical physics: material field in Mincowski space-time and curvature of Riemannian space-time itself.

Because of great success of the **General Relativity Theory** (GRT) in explanation of the existing experimental and observational facts in gravitation physics, the theory of gravitation as the theory of the field has still been deprived of the general attention and only GRT is considered in textbooks - Landau & Lifshitz 1971 [1]; Misner, Thorne & Wheeler 1973 [2]; Straumann 2013 [3] and others

However already Einstein in 1926 in work "Non-Euclidean Geometry and Physics" (“Nichtenklidische Geometrie in der Physik”) has allocated two alternative approaches to interrelation of geometry and physics. He called it Helmholtz’s and Poincare's approaches. In particular he wrote "We will accept the first (geometrical) point of view as the most answering to the current state of our knowledge". But he noted also that development in particular of the quantum theory will perhaps force to reconsider our point of view.

Field approach to gravitation has been partially developed by number of the famous physicists (e.g. Thirring 1961 [4]; Kalman 1961 [5]). A general base for relativistic quantum **Field Gravitation Theory** (FGT) was presented by Feynman in his Caltech 1962/63 lectures (see Lecture 1 – "A Field Approach to Gravitation" in Feynman, Morinigo & Wagner 1995 [6]). The gravitation phenomena in FGT are described by the relativistic quantum field which theoretically presented by the second rank symmetric tensor $\psi_{ik}$ in Minkowski space with metric tensor $\eta_{ik}$ .

A decisive step in the frame of FGT was done by Sokolov & Baryshev 1980 [7] where they founded the crucial role of the scalar (spin-0) component of the symmetric tensor field, i.e. its trace $\psi(r, t) = \eta^{ik} \psi_{ik}$ , which is the irreducible part of the symmetric tensor representation (together with the spin-2 traceless irreducible representation). The most important new aspect of the field gravitation theory is that the gravity force (Newtonian and relativistic) actually consists of the two types of fields – attraction spin-2 field and repulsion spin-0 field. Note that this fact was missed by many physicists who tried to prove the identity of the Einstein's geometrical and Feynman's relativistic quantum field descriptions of the gravitation. As it was demonstrated by Sokolov & Baryshev 1980 [7] the scalar part (trace) of the symmetric tensor potential is the true dynamical repulsive field with positive energy density, and it is not a "ghost" with negative energy density.

Recent review of the FGT results was done by Baryshev 2017 [8]. It was demonstrated that the FGT is principally different from GRT, though main really observed relativistic gravity effects have the same values in both approaches.

The reason that the intrinsic scalar field disappears in GRT but is the essential part of FGT follows from strict mathematical properties of tensors in Minkowski space. Indeed, the strict properties of the metric tensor of the Riemannian space and the general tensor rules for physical quantities in Minkowski space demand that for the sum of two quantities $\eta_{ik} + h_{ik}$ and $\eta_{ik} + \psi_{ik}$ one gets the following expressions (where $g^{ik}$ - metric tensor of the Riemannian space, $\eta_{ik}$ - metric tensor of the Minkowski space, $h_{ik}$ and $\psi_{ik}$ are small quantities of the first order):

| <u>Geometrical approach</u>: | | <u>Field approach</u>: |
|---|---|---|
| $g_{ik}(r, t) = \eta_{ik} + h_{ik}(r, t)$ | and | $f_{ik}(r, t) = \eta_{ik} + \psi_{ik}(r, t)$ |
| $g^{ik}(r, t) = \eta^{ik} - h^{ik}(r, t)$ | while | $f^{ik}(r, t) = \eta^{ik} + \psi^{ik}(r, t)$ |
| $g^{i}_{k} = \delta^{i}_{k}$ | while | $f^{i}_{k}(r, t) = \delta^{i}_{k} + \psi^{i}_{k}(r, t)$ |
| $g_{ik}\, g^{ik} = 4$ | while | $f_{ik}\, f^{ik} = 4 + 2\psi(r, t)$ |

From these relations we see that there is principle difference between geometrical and field-theoretical approaches. Indeed, in the frame of the FGT the consistent description of the sum of two tensors does not allow to change the sign for the parts. Hence tensor $f^{ik}$ cannot be the metric tensor of a Riemannian space and in the geometrical approach the scalar part of the symmetric tensor field is lost. So a "repairing" of the field approach, suggested in [2], in fact means replacing the field-theoretical approach in Minkowski space by the geometrization principle of the geometrical approach in the Riemannian space.

It leads to the new FGT predictions: that there is EMT (positive energy density of the gravitational field for both spin-2 and spin-0 parts), that besides tensor gravitational waves there are also scalar waves, and there are relativistic compact objects without horizons instead of black holes with unphysical one way surfaces.

## 2. General Relativity Theory: basic principles, main equations and predictions.

To understand the physical difference between GRT and FGT description of gravitation we start from consideration of the:

GRT basic principles:

1) ***The principle of geometrization***. General relativity is based on the principle of geometrization which indicates that all gravitational phenomena have to be described by a metrics of Riemannian space. The role of the gravitational "potential" is played by the metric tensor $\boldsymbol{g_{ik}}$ which determines the 4-interval of the corresponding Riemannian space:

$$ds^2 = g_{ik}\, dx^i dx^k \tag{1}$$

Thus, gravitation is not a material physical field in flat space-time, but is only manifestation of curved space-time.

A test particle moves along a geodesic line of the Riemannian space. Note that geodesic motion is a form of the equivalence principle, which actually has many "non-equivalent" formulations like universality of free fall or philosophical equivalence of the inertial reference frames to the reference frames accelerated by homogeneous gravity field. Equivalence principle played an important role when general relativity was born, while now the basic principle is the principle of geometrization, having clear mathematical formulation.

2) ***The principle of least action***. The field equations in Einstein's GRT are derived from the principle of the least action at a variation of a metric tensor $\boldsymbol{g_{ik}}$ in action $\boldsymbol{S}$ (matter + gravitational field). It is important to note that instead of three parts (field-interaction-matter of full action in the standard field theory) here full action contains only two parts:

$$S = S_{(m)} + S_{(g)} = \frac{1}{c}\int\left(\Lambda_{(m)} + \Lambda_{(g)}\right)\sqrt{-g}\, d\Omega \tag{2}$$

There is no Lagrangian function for interaction because in GRT gravitational interaction isn't considered while interaction Lagrangian exists for other physical fields, which is contained in the interaction part $\boldsymbol{S_{(int)}}$.

Basic equations of general relativity:

1) ***Einstein's field equations***. Variation $\boldsymbol{\delta g_{ik}}$, with restriction $\boldsymbol{g_{ik}g^{ik}} \equiv 4$ gives from $\boldsymbol{\delta(S_{(m)} + S_{(g)}) = 0}$ the following field equations:

$$\boldsymbol{\mathfrak{R}^{ik} - \frac{1}{2}\, g^{ik}\, \mathfrak{R} = \frac{8\pi G}{c^4}\, T^{ik}_{(m)}} \tag{3}$$

where $\boldsymbol{\mathfrak{R}_{ik}}$ is the Ricci tensor, $\boldsymbol{T^{ik}_{(m)}}$ is the energy-momentum tensor (EMT) of the matter.

Note that $\boldsymbol{T^{ik}_{(m)}}$ does not contain the energy-momentum tensor of the gravity field itself, because gravitation is not a material field in General Relativity (as also discussed below).

2) ***The equation of motion of test particles***. A mathematical consequence of the field equations (3) is that due to Bianchi identity the covariant derivative of the left side equals

zero, so for the right side we also have

$$T^{ik}_{(m)\,;i} = 0 \tag{4}$$

This continuity equation also gives the equations of motion for a considered matter. It implies the geodesic equation of motion for a test particle:

$$\boldsymbol{\frac{du^i}{ds} = \Gamma^i_{kl}\, u^k u^l} \tag{5}$$

$\boldsymbol{u^i = dx^i/ds}$ is the 4-velocity of the particle and $\boldsymbol{\Gamma^i_{kl}}$ is the Christoffel symbol.

<u>Major predictions for experiments/observations are:</u>

All predictions of the General Relativity (both for weak and for strong fields) are derived from Einstein's field equations and the equations of motion.

The classical weak gravity effects have been tested with accuracy of about 0.1-1%. Among these experimentally verified effects are:

- Universality of free fall for non-rotating bodies,
- Deflection of light by massive bodies,
- Gravitational frequency-shift,
- Time delay of light signals,
- Perihelion shift of a planet,
- Lense-Thirring effect,
- Geodetic precession of a gyroscope,
- The emission and detection of the quadrupole gravitational waves,

The fundamental prediction of GRT for the strong gravity is:

- Existence of the event horizon and singularity of Black Holes.

However in GRT there are also conceptual problems. The so called "energy problem" can be demonstrated with the simplest case of a spherically symmetric weak static gravity field. For instance, in harmonic coordinates the Landau-Lifshiz [1] symmetric pseudo-tensor gives negative energy density of the static spherically symmetric gravity field

$$\varepsilon_{(g)}\,(\boldsymbol{r}) = t^{00}_{LL}(\boldsymbol{r}) = -\frac{7}{8\pi G}(\nabla\varphi_N)^2 \tag{6}$$

The "final" energy-momentum tensor of the gravity field, which was derived by Grishchuk, Petrov & Popova [9], also has a negative energy density of the weak static field:

$$t^{00}_{GPP}(\boldsymbol{r}) = -\frac{11}{8\pi G}(\nabla\varphi_N)^2 \tag{7}$$

while Einstein's pseudo-tensor gives:

$$t^{00}_{E}(\boldsymbol{r}) = +\frac{1}{8\pi G}(\nabla\varphi_N)^2 \tag{8}$$

Hence, according to the LL-pseudo-tensor and the GPP-tensor the energy density of the static gravitational field is negative, which conflicts with the quantum field theories of other fundamental interactions. We note also that the traces of all these EMPTs do not vanish for static fields, while it should be for massless fields.

## 3. Relativistic Quantum Field Gravitation Theory: basic principles, main equations and predictions.

In Sec.1 we have emphasized that the field gravitation theory has its roots in papers by Birkhoff, Thirring, Kalman, and was formulated as a relativistic quantum field by Feynman [6].

Feynman [6] has shown that gravitational interaction can be described as the interaction of matter with the field of a symmetric tensor of the second rank in Minkowski's space based on a Lagrangian formalism of the field theory. He discussed a quantum field approach to the gravity just as the next fundamental physical interaction and claimed that “the geometrical interpretation is not really necessary or essential to physics” ([6] p. 113).

In the frame of the field gravitation theory the crucial role of the intrinsic scalar part (the trace $\psi(r, t) = \eta_{ik}\psi^{ik}$) of the reducible symmetric tensor potentials $\psi_{ik}(r, t)$ was discovered and studied by Sokolov & Baryshev 1980 [7] (recent review in Baryshev 2017 [8].

Basic principles of Relativistic Quantum Field Gravitation Theory include:

• the inertial reference frames and Minkowski space with metric $\eta_{ik}$;

• the reducible symmetric second rank tensor potential $\psi^{ik}{}_{(r,t)}$ and especially its trace $\psi_{(r,t)} = \eta_{ik}\,\psi^{ik}$ describe gravitational interaction;

• the Lagrangian formalism and Stationary Action principle:

$$S = S_{(m)} + S_{(int)} + S_{(g)} = \frac{1}{c}\int\left(\Lambda_{(m)} + \Lambda_{(int)} + \Lambda_{(g)}\right)\sqrt{-g}\,d\Omega \qquad (9)$$

where

$$\Lambda_{(int)} = -\frac{1}{c^2}\,\psi_{ik}T^{ik} \qquad (10)$$

• the principle of consistent iterations;

• the universality of gravitational interaction $\Lambda_{(int)}$;

• the conservation law of the energy-momentum;

• the gauge invariance of the linear field equations;

• the positive localizable energy density and zero trace of the gravity field EMT;

• the quanta of the field energy as the mediators of the gravity force;

• the uncertainty principle and other quantum postulates of the QFT.

Basic equations of the Field Gravity Theory:

1) ***FGT field equations***.

Using the variation principle to obtain the field equations from the action (9) one must assume that the sources $T_{ik}$ of the field are fixed (or the motion of the matter given) and vary only the potentials $\psi_{ik}$ (serving as the coordinates of the system). On the other hand, to find the equations of motion of the matter in the field, one should assume the field to be given and vary the trajectory of the particle (matter). So keeping the total EMT of matter in (10) fixed and varying $\delta\psi_{ik}$ in (9) we get the following field equations (see [8]):

$$-\psi_{l}^{ik,l} + \psi_{l}^{il,k} + \psi_{l}^{kl.i} - \psi^{,ik} - \eta^{ik}\psi^{lm}_{,lm} + \eta^{ik}\psi_{l}^{,l} = \frac{8\pi G}{c^2}T^{ik} \qquad (11)$$

The trace of the field equations (11) gives the scalar equation for generating the scalar part of the symmetric second rank tensor – its trace ***ψ(r, t)***, in the form

$$-2\psi_{l}^{,l} + 2\psi_{,lm}^{lm} = \frac{8\pi G}{c^2} T \tag{12}$$

An important conceptual difference between the coordinate’s transformation in GRT and the gauge transformation of the gravitational potentials in FGT is that these gauge transformations are performed in a fixed inertial reference frame. The gauge freedom allows one to put four additional conditions on the potentials, in particular a Lorentz invariant gauge – the Hilbert-Lorentz gauge:

$$\psi_{,k}^{ik} = \frac{1}{2}\,\psi^{,i} \tag{13}$$

With the gauge (13) the field equations get the form of wave equation:

$$\left(\nabla - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\psi^{ik} = \frac{8\pi G}{c^2}\left[T^{ik} - \frac{1}{2}\,\eta^{ik}\,T\right] \tag{14}$$

which describes two types of waves: first, the spin-2 traceless irreducible representation $\psi_{\{2\}}^{ik} = \psi^{ik} - (1/4)\,\psi(r,t)\,\eta^{ik}$ and second, the scalar (spin-0) component of the symmetric tensor field, i.e. its trace $\psi(r,t) = \eta^{ik}\,\psi_{ik}$, which is the second irreducible part of the symmetric tensor representation.

***2) Equation of motion:***

Equation of motion for test particles in the field gravity theory ([8]):

$$A_k^i\,\frac{d(m_0 u^k)}{ds} = -m_0 B_{kl}^i u^k u^l \tag{15}$$

where $m_o u_k = p_k$ is the 4-momentum of the particle, and

$$A_k^i = \left(1 - \frac{1}{c^2}\,\psi_{ln} u^l u^n\right)\eta_k^i - \frac{2}{c^2}\,\psi_{kn} u^n u^i + \frac{2}{c^2}\,\psi_k^i \tag{16}$$

$$B_{kl}^i = \frac{2}{c^2}\,\psi_{k,l}^i - \frac{1}{c^2}\,\psi_{kl}^{,i} - \frac{1}{c^2}\,\psi_{kl,n} u^n u^i \tag{17}$$

Relativistic gravity experiments/observations in FGT:

- Universality of free fall for non-rotating bodies,
- The deflection of light by massive bodies,
- TheGravitational frequency-shift,
- The time delay of light signals,
- The perihelion shift of a planet,
- The Lense-Thirring effect,
- The geodetic precession of a gyroscope,
- The quadrupole spin-2 and monopole spin-0 gravitational radiation

For the strong gravitational field the fundamental prediction is:

- Relativistic Compact Objects without horizon, instead of Black Holes.

In this theory there is no problem with energy of gravitational field. In calculations Oschepkov 1995 [10] it is shown that energy density of static spherically symmetric gravitational field not only is positive for canonical EMT, but also for Gilbert's EMT:

$$T^{00}_{(canon)} = T^{00}_{(Gilbert)} = +\frac{1}{8\pi G}(\nabla\varphi_N)^2 \qquad (18)$$

In the following report “Gravitation theory in multimessenger astronomy II: crucial observational tests based on GW and optical observations” we consider applications of metric and field gravitation theories for interpretations of astrophysical observations.

## References


[1] Landau, L.D., Lifshitz, E.M., 1971, The Classical Theory of Fields. (Pergamon, Oxford) (1971)

[2] Misner, C., Thorne, K., Wheeler, J. Gravitation. Freeman, San Francisco (1973)

[3] Straumann N., General Relativity. Springer (2013)

[4] Thirring W. E., An alternative approach to the theory of gravitation. Ann. of Phys. 16, 96 (1961)

[5] Kalman G., Lagrangian formalism in relativistic dynamics. Phys.Rev. 123, 384 (1961)

[6] Feynman R., Morinigo F., Wagner W., Feynman Lectures on Gravitation, Addison-Wesley Publ. Comp. (1995)

[7] Sokolov V.V., Baryshev Yu.V., Field-theoretical approach to gravitation: energy-momentum tensor of the field, Gravitation and Relativity Theory, Kazan State University, vyp.17, 34 (1980)

[8] Baryshev Yu.V., Foundation of relativistic astrophysics: Curvature of Riemannian Space versus Relativistic Quantum Field in Minkowski Space. Arxiv 1702.02020 (2017).

[9] Grischuk, L.P., Petrov, A.N., Popova, A.D., Exact theory of the (Einstein) gravitational field in an arbitrary background space-time, Commun. Math. Phys., 94, 379 (1984)

[10] Oschepkov S.A, Energy of static spherical-symmetrical field in Gravidynamics, Gravitation, Vol. 1, Issue 1, 1995, pp. 36-40. (http://astro.okis.ru/files/1/7/2/172962/04_oshepkov_en.pdf )