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Magnetic Fields of M–Dwarfs from the Molecular and Atomic Diagnostics

Shulyak D.[1], Reiners A.[1], Wende S.[1], Kochukhov O.[2], Piskunov N.[2], Seifahrt A.[3]

[1] Institute of Astrophysics, Göttingen University, Göttingen, Germany
[2] Department of Physics and Astronomy, Uppsala University, Uppsala, Sweden
[3] Department of Physics, University of California, Davis, USA

Abstract. Strong surface magnetic fields are frequently found in the spectra of M–dwarfs, with the mean intensities on the order of a few thousand Gauss — three orders of magnitude higher than the mean surface magnetic field of the Sun. The appearance of such fields in both partially and fully convective M–dwarfs provides strong constraints on the theoretical models of stellar magnetism. The accurate estimates of the magnetic field intensity and geometry in these cool objects, however, is strongly limited to our ability to simulate the Zeeman effect in molecular lines. Here we present the first quantitative results of modelling and analysis of the magnetic fields in selected M–dwarfs in FeH Wing–Ford $F^4\Delta$–$X^4\Delta$ transitions and some strong atomic lines from polarised radiative transfer.

Key words: stellar atmospheres – low–mass stars – magnetic fields

1 Overview

Magnetic fields in non–degenerate stars are found all across the Hertzsprung–Russell diagram, from hot high–luminous stars down to cool and ultra–cool dwarfs (see, for example, the review by Donati & Landstreet, 2009, and references therein). These fields span a wide range of intensity and geometry, thus providing strong experimental ground for the modern theories of stellar magnetism.

In particular, the characterisation of magnetic fields in cool low–mass M–stars is of high interest because a) these stars contribute up to 70% by number of the total stellar population in our galaxy; b) these stars often show dynamo–generated activity in their atmospheres similar to that of the Sun (i. e. with flares seen as strong emission in the X–ray, Hα, Ca II H&K lines, see Berdyugina, 2005), however, objects later than M3.5 ($T_{\rm eff} \approx 3400$ K, $M < 0.35 M_{\odot}$) are believed to become fully convective, therefore different dynamo mechanisms need to be involved to explain their fields; c) these stars are promising targets to search for Earth–like planets inside their habitable zones, and thus knowledge of the magnetic activity of a parent star is potentially important for the climate modelling of these distant Earths.

Measurements of the magnetic fields in cool stars relies on polarimetric observations and analysis of Zeeman broadening of spectral lines. Even with modern instruments polarimetric observations remain a very challenging task as many hours of integration time are usually needed to achieve sufficient S/N even for brightest M–dwarfs. Recent systematic studies of magnetic fields in a number of M–dwarfs using the observations in Stokes I and V parameters led to the discovery of rather strong magnetic fields (up to several kG) on the surface of M–dwarfs and made possible to estimate their topologies. In particular, using Zeeman Doppler Imaging (ZDI) and Least Squares Deconvolution

(LSD) technique, Donati et al. (2008) and Morin et al. (2008) found a clear transition in the magnetic field topologies between partly and fully convective stars, with later tending to host strong, poloidal, mostly axisymmetric fields contrary to much weaker, non–axisymmetric fields with a strong toroidal component for the former. In contrast, a most recent investigation revealed a presence of M–dwarfs with strong toroidal non–axisymmetric fields among fully convective objects, and thus no clear transition between these two groups of M–stars seems to exist any more (Morin et al., 2010). One should not forget that the above analysis is based on the Stokes V measurements, which are sensitive only to large scale field components. This could potentially result in missing a critical amount of magnetic flux, as the Stokes V signal could cancel out when observing large–scale axisymmetric fields. In addition, LSD technique itself has a number of assumptions which do not necessarily reflect the behaviour of individual spectral lines (see, for example, Kochukhov et al., 2010).

The analysis of Zeeman broadening in individual lines was also successfully applied to the spectra of M–dwarfs (Saar, 2001). The Zeeman effect is sensitive to the surface averaged magnetic field modulus and thus naturally gives information about the true magnetic field flux. Strong fields up to ≈ 4 kG were then reported for some M–dwarfs based on the relative analysis of magnetically sensitive atomic lines (Johns–Krull & Valenti, 1996, 2000). For dwarfs cooler than mid–M, atomic lines decay rapidly and are lost in the forest of molecular features. As a result, the search for alternatives ended up with molecular lines of FeH Wing–Ford $F^4\Delta - X^4\Delta$ transitions around $0.99\,\mu$m (Valenti et al., 2001; Reiners & Basri, 2006). Some of these lines do show a strong magnetic sensitivity, as seen, for instance, in the sunspot spectra (Wallace et al., 1998).

The modelling of the Zeeman effect in FeH lines though faces a great difficulty: most lines are formed in the intermediate Hund's case, the theoretical description of which is based on certain approximations. The main problem is connected with the Born–Oppenheimer approximation, which was used in the theoretical descriptions of level splitting, and which assumes a clear separation between the electronic and nuclear motion in terms of energies. This approximation fails for FeH because the energy separation between the electronic states is of the order of or smaller than the energy separation between individual vibrational levels. Among recent improvements in understanding the Zeeman splitting in diatomic molecules one should mention the papers by Berdyugina & Solanki (2002) and Asensio Ramos & Trujillo Bueno (2006). However, theoretical g–factors still cannot describe correctly the broadening of a great majority of FeH lines.

A promising solution was then suggested by Reiners & Basri (2006, 2007), who estimated the magnetic fields (or, more precisely, product ($|\mathbf{B}|f$), where f is a filling factor) in a number of M–dwarfs by a simple linear interpolation between the spectral features of two reference stars with known magnetic fields, but the error bars of such an analysis stays high (≈ 1 kG).

Later, Afram et al. (2008) made use of a semi–empirical approach to estimate the Landé g–factor of FeH lines in the sunspot spectra. The authors succeeded to obtained a very good agreement with observations for selected FeH lines and presented best–fitted polynomial g–factors of upper and lower levels of corresponding transitions. A little earlier, Harrison & Brown (2008) presented the empirical g–factors for a number of FeH lines originating from levels with rotational quantum numbers $\Omega = 7/2, 5/2, 3/2$, but limited to low magnetic J–numbers. The authors also presented a way in which the effective Hamiltonian approach can still be used by modifying the electronic spin g_{S} and orbital magnetic g_{L} factors from their theoretical values. As an example, Fig. 1 illustrates a comparison between the experimental and theoretical g–factors for the lower ($X^4\Delta$) and upper ($F^4\Delta$) states of FeH transitions with rotational quantum number $\Omega = 2.5$. It is seen that the lower state tends to be close to the pure Hund's case (b) and the upper state is in the intermediate case, while theoretical predictions always give solutions between pure (a) and (b) cases, thus failing to reproduce the g–factors for the lower state. All the studies mentioned above clearly show the main problems of the modern theory of the Zeeman effect in the intermediate Hund's case, and point in the direction of combining the empirical and theoretical approaches.

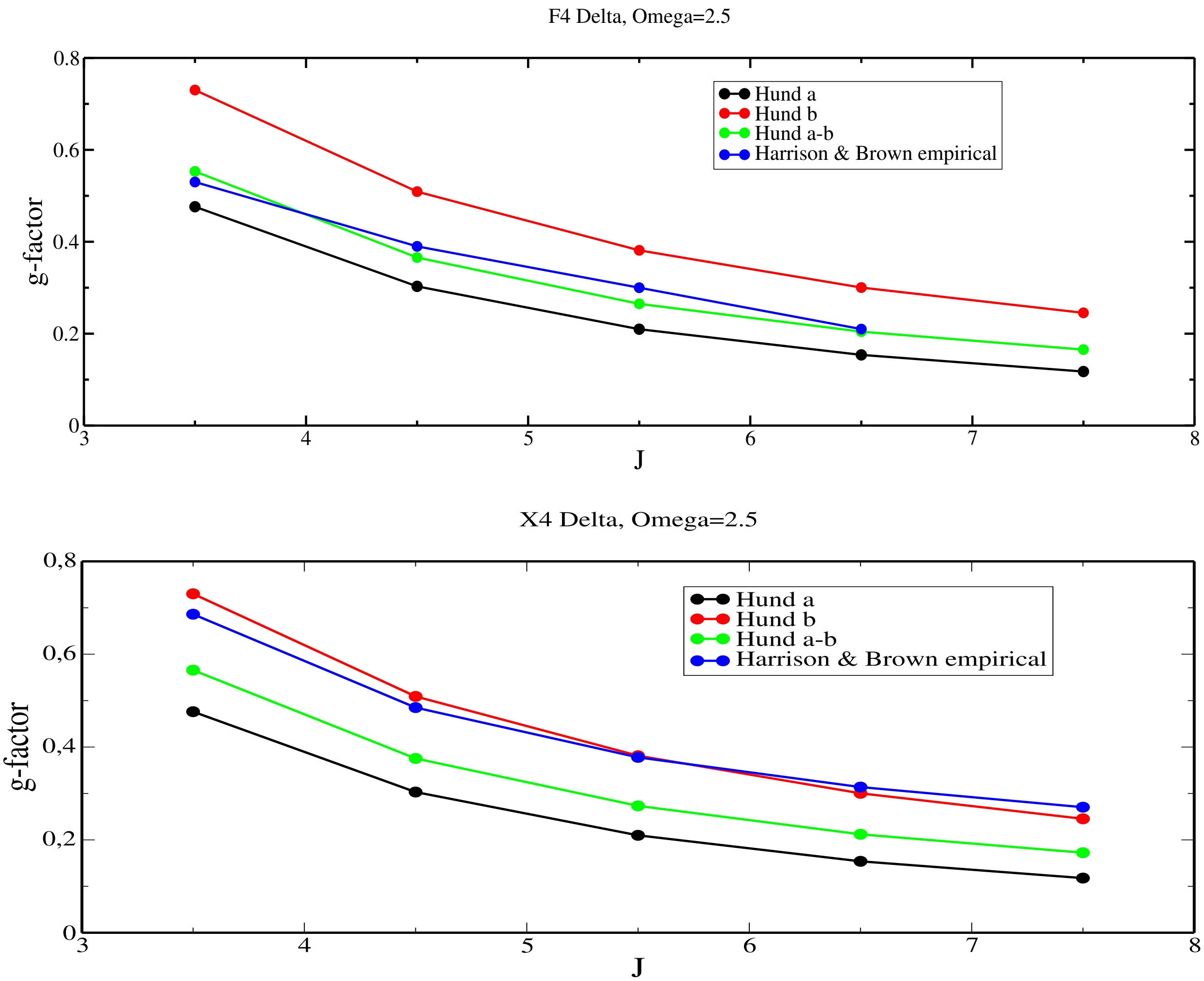

Figure 1: Comparison between observed (Harrison & Brown, 2008) and theoretical g–factors of lower ($X^4\Delta$) and upper ($F^4\Delta$) levels of FeH with rotation quantum number $\Omega = 2.5$ as a function of magnetic quantum number J. Black line — pure Hund's case (a), red line — pure Hund's case (b), green line — intermediate between pure Hund's (a) and (b) cases.

The empirical analysis of FeH lines presented in Reiners & Basri (2006, 2007) is the only estimate of the magnetic field using the molecular lines in M–dwarfs available so far. Yet the procedure employed in these studies is not physically justified and thus requires more quantitative investigations. Further analyses of FeH lines also employed the same method to measure the M–dwarf magnetic fields (e. g., Reiners et al., 2009; Reiners & Basri, 2009, 2010). It is important to realize that so far all field measurements in FeH lines are anchored in the measurement of the field strength of EV Lac, $\langle|\mathbf{B}|f\rangle = 3.9\,\mathrm{kG}$, which was carried out in a single atomic Fe line by Johns–Krull & Valenti (2000). Here we attempt to provide an independent measurement from a number of very magnetically sensitive molecular FeH lines, which are modelled based on the formalism described in Berdyugina & Solanki (2002). Our main goal is to combine the information from the available atomic and molecular diagnostics employing the direct spectrum synthesis.

2 Methods

2.1 Input Line Lists and Synthetic Spectra

In the present investigation we made use of FeH lines of the Wing–Ford band ($F^4\Delta$–$X^4\Delta$ transitions, $0.9-1\,\mu$m) which were found to be an excellent diagnostics of the magnetic field in the spectra of sunspots and cooler M–dwarfs (see, for example, Valenti et al., 2001; Reiners & Basri, 2006, 2007; Afram et al., 2007,2008; Harrison & Brown, 2008). The line list of FeH transitions and molecular constants were taken from Dulick et al. (2003)[1]. For some FeH lines it was necessary to adjust their Einstein A values to match the observations and thus obtain a consistent fit between all of them, several strong atomic lines of Ti I, and with the same model atmosphere. This is described in more detail in Wende et al. (2010).

The VALD (Vienna Atomic Line Database) was used as a source of atomic transitions (Piskunov et al., 1995; Kupka et al., 1999). We decreased the $\log(gf)$ values of Ti I 10610 Å and 10735 Å lines by 0.2 dex to match the observations of a non–active star GJ 1002. Again, this ensured a consistent fit simultaneously with other strong Ti lines using the same atmospheric parameters.

To compute synthetic spectra of the atomic and molecular lines in the magnetic field we employed the SYNMAST code (Kochukhov, 2007). The code represents an improved version of the SYNTHMAG code described by Piskunov (1999).

Model atmospheres are from the recent MARCS grid[2] (Gustafsson et al., 2008).

2.2 Molecular Zeeman Effect

In order to analyse the magnetic field through the spectra synthesis it is necessary to know the Landé g–factors of the upper and lower levels of a particular molecular transition. Important to us, in case of diatomic molecules the simple analytical expressions for the g–factors can only be obtained in pure (a) and (b) cases. Sad but true, as stated in Berdyugina & Solanki (2002), the lines of FeH of Wing–Ford band exhibit splitting, which is in most cases intermediate between pure Hund's (a) and (b) and which is not trivial to treat both theoretically and numerically. In the present work, we implement the numerical libraries from the MZL (Molecular Zeeman Library) package originally written by B. Leroy (Leroy, 2004), and adopted for the particular case of FeH. The MZL is a collection of routines for computing the Zeeman effect in diatomic molecules, and it contains the complete physics of pure and intermediate Hund's cases presented in Berdyugina & Solanki (2002). For the atomic lines the g–factors were directly extracted from VALD.

3 Results

3.1 Atomic Lines in Sunspot Spectra

Before presenting the results of the M–star spectra analysis, it is necessary to verify our methods through observations of some standard star where both atomic and molecular lines can be seen simultaneously. A sunspot spectrum is probably the only trustworthy data source in this regard because a) the temperature inside a spot is still hot enough to see strong unblended atomic and FeH lines and b) very high–resolution and S/N observations are available. We thus made use of an umbral spectrum from Wallace et al. (1998). They also derived the magnetic field intensity $|\mathbf{B}|=2.7$ kG. Our fit to the atomic lines (mostly Fe, Ti, and Cr) in the range of $9800-10800$ Å also suggests a field intensity $|\mathbf{B}|=2.7$ kG (model atmosphere with $T_{\rm eff}=4000$ K). As an example, Fig. 2 illustrates theoretical fits for some selected atomic lines.

[1] http://bernath.uwaterloo.ca/FeH/
[2] http://marcs.astro.uu.se

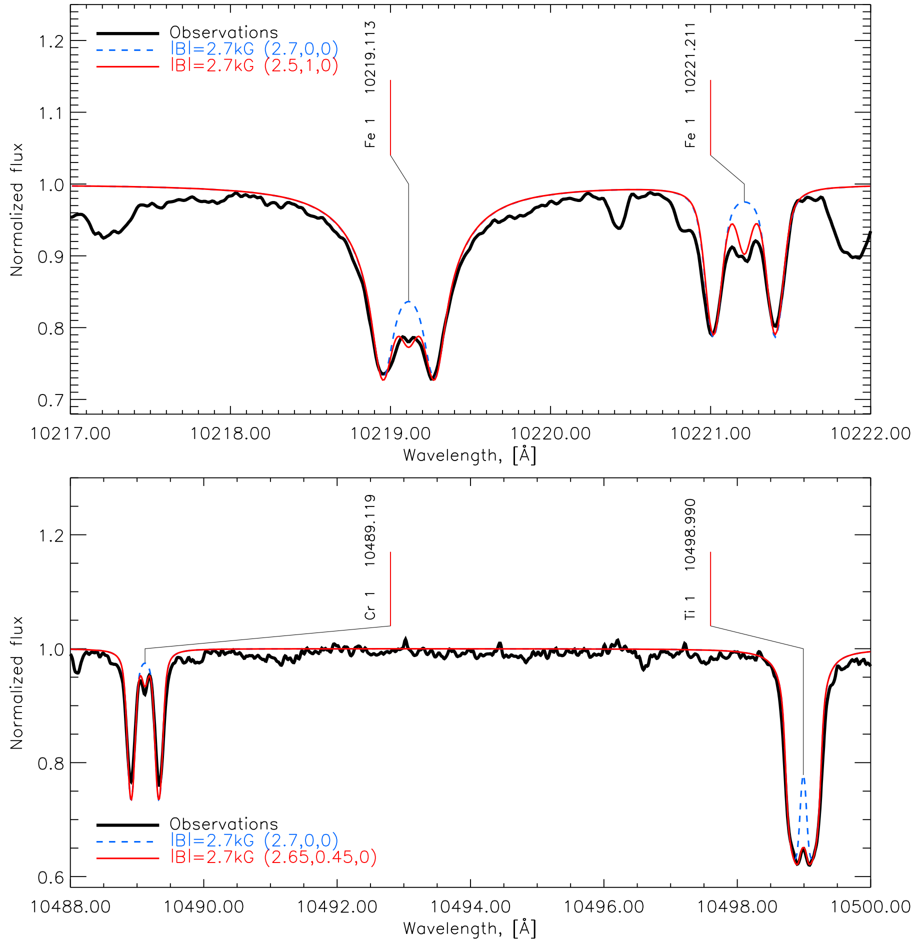

Figure 2: Observed and predicted spectra of a sunspot. Theoretical computations are shown for two magnetic field geometries: purely radial $(B_r, 0, 0)$ and with a horizontal contribution $(B_r, B_m, 0)$ (see legends on individual plots). Wavelengths are in vacuum.

3.2 Landé Factors of FeH Lines

As noted above, Wing–Ford lines of FeH are mostly formed in the intermediate Hund's case, which makes it difficult to accurately predict the Zeeman patterns of the respective states. Analytically, the type of splitting can be estimated via the analysis of spin–orbit and rotational constants $Y = |A_v|/B_v$: if $Y \gg J(J+1)$ when an approximation of Hund's case (a) is valid, and Hund's case (b) otherwise (Herzberg, 1950).

Using the MZL library and sunspot spectra we tried to compute the theoretical g–factors for all the lines under the condition that the resulting Zeeman patterns provide correct broadening of individual FeH lines in the sunspot spectra. In general, we find that the intermediate case (with its present treatment in MZL) is a good approximation if (l — lower, u — upper states)

1. $\Omega_l = 0.5$
2. $\Omega_{l \text{ or } u} \leq 2.5$ and $3Y > J(J+1)$ for the P and Q branches
3. $\Omega_{l \text{ and } u} = 2.5$ and $5Y > J(J+1)$ for the R branch.

For the rest of transitions the assumption of Hund's case (a) for the upper level and Hund's case (b) for the lower level provide reasonable results, especially for the transitions with $\Omega_{l \text{ and } u} = 3.5$.

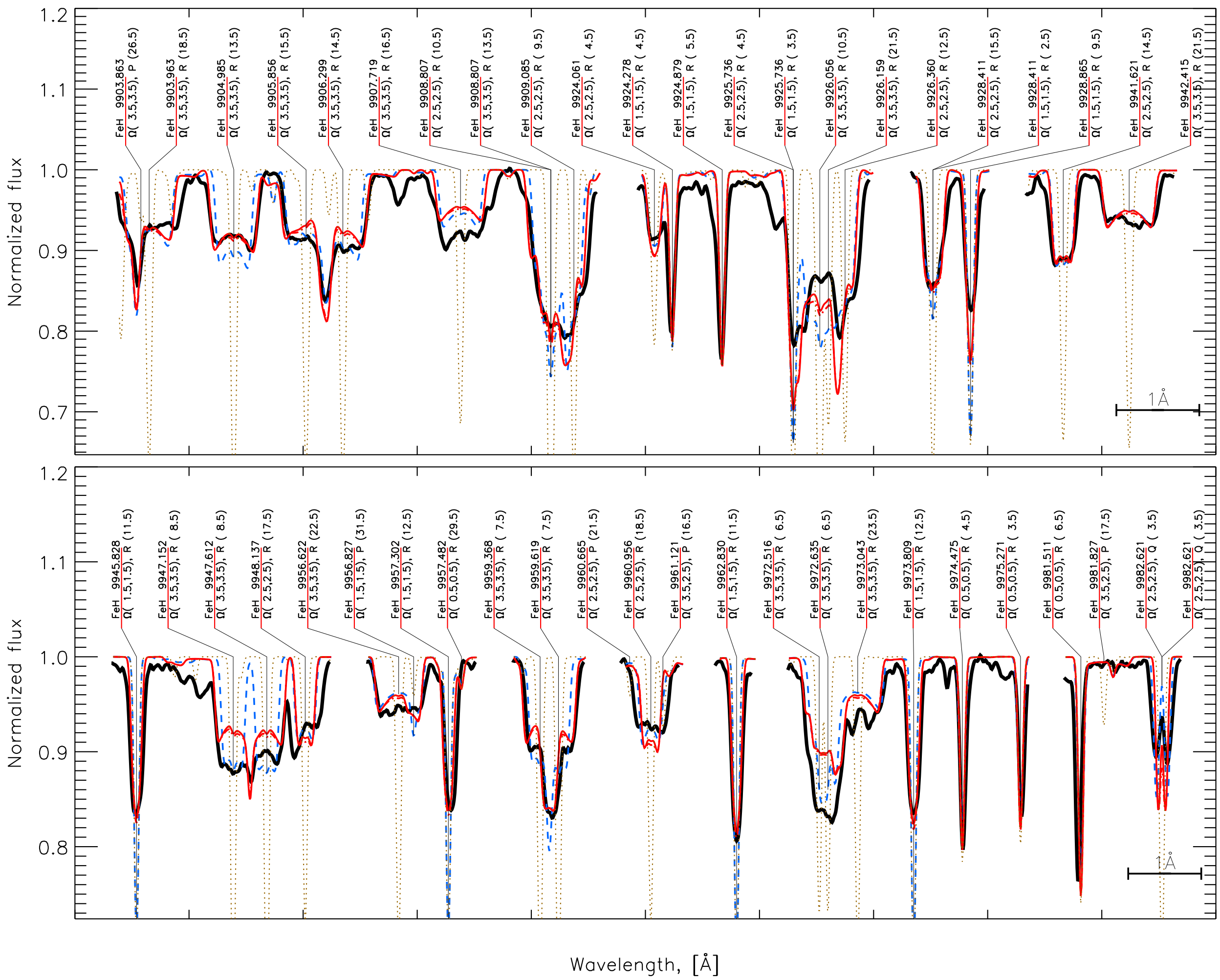

Figure 3: Comparison between observed and theoretical sunspot spectra in the selected regions of FeH transitions. The blue dashed line represents the best–fitted g–factor from Afram et al. (2008), the red solid line — calculations with MZL library; both with a purely radial field of $\mathbf{B} = (2.7, 0, 0)$ kG. The red dash–dotted line — synthetic spectra accounted for the horizontal field component $\mathbf{B} = (2.5, 1, 0)$ kG (hardly seen in the figure, coincides with the red solid line), the brown dotted line is the zero–field spectrum. The labels over the lines indicate their central wavelengths, omegas of lower and upper states (in brackets), branch, and the J–number of the lower state (in brackets). Wavelengths are in the vacuum.

Figure 3 illustrates a comparison between the best–fitted g–factor from Afram et al. (2008) and our calculation for some selected lines in the $9900 - 10000$ Å region. Note that apart from the figures shown in Afram et al. (2008), we did not make an attempt of correcting the theoretical spectra, i. e. no filling factors were applied. The model atmosphere and field intensity are the same as determined previously from the metallic line spectra. From Fig. 3 it is obvious that there is in general a good agreement between our calculation and the g–factors from Afram et al. (2008). Still, the discrepancy between the two calculations is found for the low omega R–branch lines like FeH 9945 Å, 9962 Å, etc., which indeed show splitting closer to pure Hund's cases. Even though we succeeded well enough to fit the width of the observed lines with the same field of $|\mathbf{B}| = 2.7$ kG previously derived from atomic lines, this cannot be judged to be more physical, though until new improvements in the theoretical description of the intermediate case will become available.

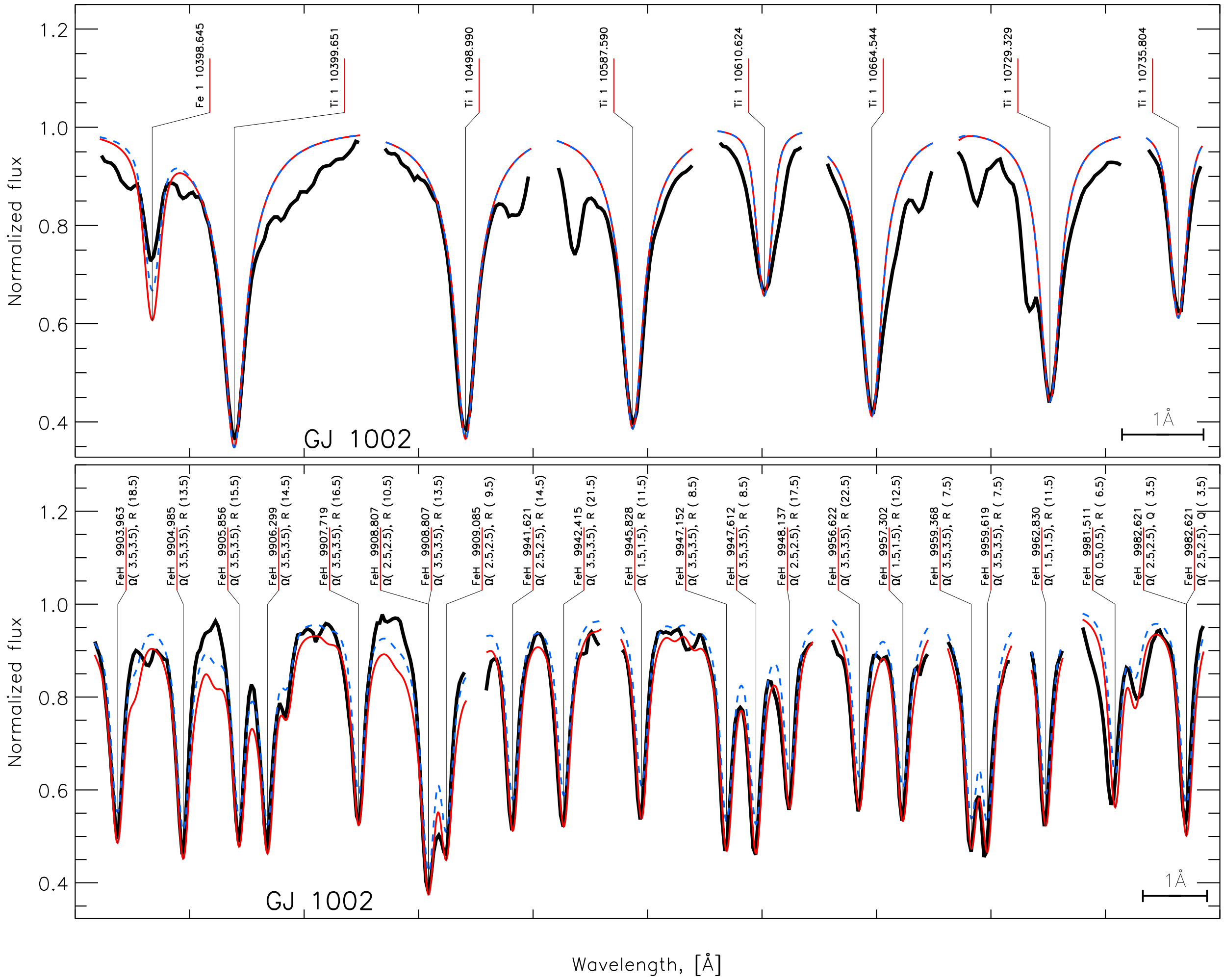

Figure 4: Observed and predicted spectra of M5.5 dwarf GJ 1002 in Ti I (*upper plot*) and selected FeH lines (*lower plot*). Model parameters: $T_{\rm eff} = 3100$ K, $\log g = 5.0$, $[{\rm M/H}] = 0.0$, $v \sin i = 2.5$ km/s. Observations are shown by the thick solid line. Theoretical spectra were computed assuming $\varepsilon_{\rm Fe} = -4.37$ (red solid line) and $\varepsilon_{\rm Fe} = -4.59$ (blue dashed line), respectively. Wavelengths are in the vacuum.

3.3 Testing Formation of FeH Lines in the Atmosphere of Non–Magnetic GJ 1002

In cool atmospheres, the van der Waals constant is one of the most important broadening mechanisms. For FeH, there are no theoretical or laboratory measurements of broadening constants available. We thus checked the spectra of FeH in the atmosphere of the non–magnetic M5.5 star GJ 1002 and found that the classical van der Waals constant (see Gray, 1992) used in Synmast must be increased by a factor of 3.5 to satisfactorily match the profiles of FeH lines. A model atmosphere with $T_{\rm eff} = 3100$ K, $\log g = 5$, $[{\rm M/H}] = 0.0$, and $v \sin i = 2.5$ km/s can fit strong Ti I lines in the region of $10300 - 10800$ Å, as shown in Fig. 4. Observations are from the CRIRES ($R = 100\,000$).

3.4 Example of an Active M4.5 dwarf GJ 1224

GJ 1224 is an active M4.5 dwarf, the previous attempts to measure its magnetic field resulted in $(|\mathbf{B}|f) = 2.7$ kG (Reiners & Basri, 2007). An analysis of the magnetically insensitive FeH lines and

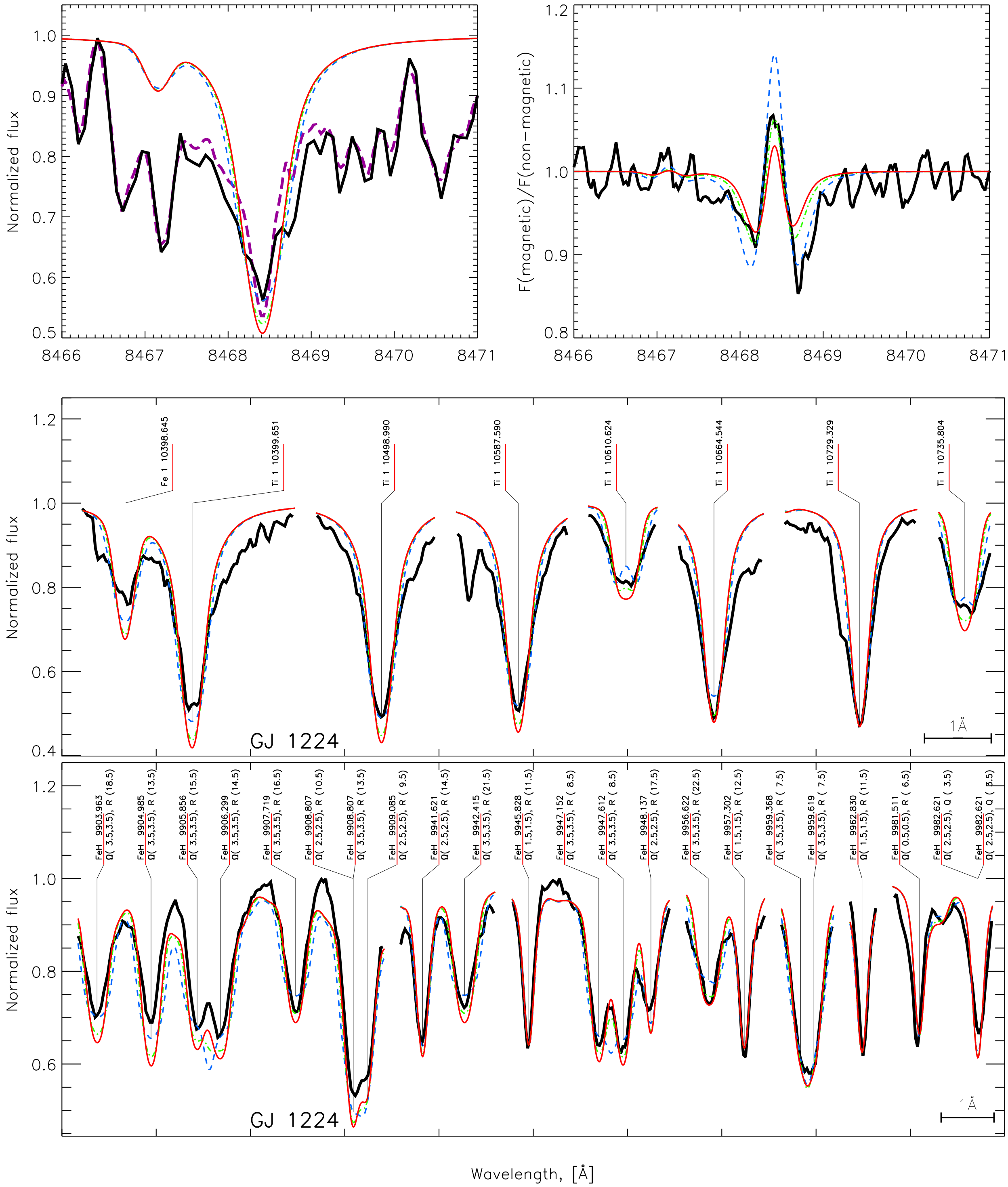

Figure 5: Comparison between the observed and theoretical spectra of the M4.5 dwarf GJ 1224.
Upper panel: observed and calculated spectra at the Fe I 8468 Å line (*left plot*) and the ratio between the corresponding magnetic and non–magnetic spectra (*right plot*). The thick solid line — observations of GJ 1224, the violet long–dashed line — observations of inactive LHS 337.
Middle and lower panels: same as in Fig. 4. Model parameters: $T_{\rm eff}=3200$ K, $\log g=5.0$, $\varepsilon_{\rm Fe}=-4.45$, $v\sin i=3$ km/s. The thick solid line — observations, the blue dashed line — $\mathbf{B}=(2.7,0,0)$ kG, the green dash-dotted line — $\mathbf{B}=(2,0,0)$ kG, the red solid line — $\mathbf{B}=(1.7,0,0)$ kG. Wavelengths are in air for the upper plots and in the vacuum for the middle and lower plots.

Ti I 10729.3 Å line suggested $T_{\rm eff} = 3200$ K, $\log g = 5.0$ and $v \sin i = 3$ km/s under the assumption of solar Ti abundances. We found it necessary to slightly increase the Fe abundance to $\varepsilon_{\rm Fe} = -4.45$ to match the magnetically insensitive FeH lines.

The upper plot of Fig. 5 compares the observed and predicted spectra of the Fe I 8468 Å line, which was previously used by Johns–Krull & Valenti (2000) to estimate the magnetic field in a number of M–dwarfs. There is a general difficulty of measuring the magnetic field strength from this line though because of the heavy blending by TiO. Therefore, following Johns–Krull & Valenti (2000), we tried to investigate the relative line intensity by dividing the spectra of GJ 1224 by the spectra of a non–magnetic M–dwarf LHS 337, which has the same or very similar spectral type. The resulting profiles are shown in the right upper panel of Fig. 5. The field of 2.7 kG can fit the slope of the red wing of the residual, but a weaker field of about 2 kG is required for the blue wing of the line. This discrepancy is likely due to the relatively low quality of the data and strong blending by TiO.

The middle and lower plots of the Fig. 5 illustrate the fit to the same set of Ti and FeH lines as performed for the non–magnetic GJ 1002 assuming different magnetic field intensities. There are several things to note. First of all, it is impossible to fit the cores of magnetic–sensitive Ti lines with any field geometry and intensity. Fields larger than 2 kG result in cores that are too wide to be observed. Inversely, using $|\mathbf{B}| \leq 2$ kG allows us to obtain a reasonable fit to the width of line cores, but the predicted central depths are too strong for some lines. This behaviour is broken for the Ti 10664.5 Å line, for which $|\mathbf{B}| = 2$ kG and $|\mathbf{B}| = 1.7$ kG provide a good fit, and the Ti 10610 Å line the core of which can only be described by $|\mathbf{B}| = 2$ kG. Another line at 10735 Å seems to point in the direction of $|\mathbf{B}| \approx 2.7$ kG, but the data quality at the red end of the spectra is poor and it is impossible to draw accurate conclusions.

Extending this analysis to the FeH lines brings stronger constraints for the magnetic field intensity, as shown in the lower panel of Fig. 5. In particular, magnetically sensitive lines such as FeH 9905 Å, 9906 Å, 9942 Å, and 9959 Å clearly point to the field modulus $|\mathbf{B}| \leq 2$ kG. The fact that the widths of these lines are well reproduced in the sunspot spectra (see Fig. 3) allows us to consider them as important indicators of the mean field intensity. In particular, increasing $|\mathbf{B}|$ results in the appearance of the characteristic feature owing to the crossed σ–components of the two FeH lines at 9906 Å. Overlaid, these components give rise to the absorption feature *which is not seen in the observed spectra.* Consequently, weaker fields are needed to keep these lines separated. Changing the magnetic field geometry by varying the strength of the horizontal field components does not help to disable this feature: in our computations this is only possible with $1.7 \leq |\mathbf{B}| \leq 2$ kG, preferably with $|\mathbf{B}| \approx 1.7$ kG.

Table 1 gathers the main results of the present study including several other M–dwarfs which were analysed applying the same approach. We refer the interested reader to the work of Shulyak et al. (2010) for more details. In Table 1, average surface magnetic fields resulting from the analysis of atomic and FeH lines are shown separately.

4 Summary

In this study we made an attempt of using the up-to-date knowledge of the molecular Zeeman effect, modern software for the magnetic spectra synthesis, and molecular lines data to develop an approach of modelling the Zeeman splitting in FeH lines of the Wing–Ford $F^4\Delta - X^4\Delta$ band. This approach was then applied to measure the magnetic fields in selected M–dwarfs for which the observations in both atomic and FeH lines are available. The main results of the present work can be summarised as follows:

1. Our results of magnetic field strengths derived from FeH lines are 15 − 30% lower than the results presented in Reiners & Basri (2007), which are based on the atomic line analysis

Table 1: Atmospheric parameters of investigated M–dwarfs

Name	Spectral type	$T_{\rm eff}$ (K)	$\upsilon \sin i$ (km/s)	$\lvert\mathbf{B}\rvert_m$ (kG)		$(\lvert\mathbf{B}\rvert f)$ (kG)
				atoms	FeH	
Sunspot	—	4000	0.0	2.7	2.7	$2.7^{(1)}$
GJ 1002	M5.5	3100	2.5	0	0	—
GJ 1224	M4.5	3200	3.0	≈ 2	$1.7 - 2$	$2.7^{(2)}$
YZ Cmi	M4.5	3300	5.0	$3 - 4$	$3 - 3.5$	$>3.9^{(2)}$
EV Lac	M3.5	3400	1.0	$3 - 4$	$3 - 3.5$	$\sim 3.9^{(3)}$
AD Leo	M3.5	3400	3.0	$2 - 3$	$2 - 2.5$	$\sim 2.9^{(2)}$

$\lvert\mathbf{B}\rvert_m$ — mean surface magnetic field
$(\lvert\mathbf{B}\rvert f)$ — results of previous investigations

(1) — Wallace et al. (1998)
(2) — Reiners & Basri (2007), scaled from (3)
(3) — Johns–Krull & Valenti (1996)

scaled from Johns–Krull & Valenti (2000).

2. We confirm a strong magnetic field found in YZ CMi, EV Lac, and AD Leo but with an intensity which is likely to be 500 G (or more) weaker. Unfortunately, the poor quality of the data forbids more quantitative conclusions.

3. An analysis of atomic and FeH lines in the spectra of GJ 1224 points in the direction of 1.7 – 2 kG averaged magnetic field, which is lower than 2.7 kG previously reported (Reiners & Basri, 2007).

4. The estimates of the magnetic field modulus from the Fe I 8468 Å line seem to be systematically higher than those from FeH lines. This, however, should be taken with caution due to the unknowns associated with the quality of the data and atmospheric parameters used.

5. With routines provided by the Molecular Zeeman Library we developed an algorithm for calculating the Landé g–factors of FeH states: the choice of the Hund's cases (a), (b) or intermediate follows from the consideration of quantum numbers of individual states. The test computations of the sunspot spectra showed a good agreement with observations and with calculations based on the best–fit g–factors from Afram et al. (2008).

6. To distinguish between different magnetic field geometries, higher quality observations are required for the analysis of FeH spectra ($S/N > 100$).

7. To exclude possible selection effects caused by the variable field associated with the magnetic spot(s), and to provide a complete characterisation of the magnetic field intensity and geometry, time-resolved observations are highly desired. These observations, which would ideally provide spectra in all four Stokes parameters as well, remain highly difficult even for the brightest M–dwarfs since they require many hours of signal integration time to achieve the desired S/N even with modern astronomical instrumentation.

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